RD Sharma Solutions Class 11 Maths Chapter 22 Ex 22.1

Brief Review of Cartesian System of Rectangular co-ordinates Ex 22.1 Q1

It is given that O is the origin.
Then,

$$OQ^2 = x_2^2 + y_2^2,$$

$$OP^2 = x_1^2 + y_1^2$$

and, $PQ^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$

Using cosine fromula in □OPQ, we have

$$PQ^2 = OP^2 + OQ^2 - 2.(OP)(OQ)\cos\alpha$$

$$\Rightarrow \quad (x_2 - x_1)^2 + (y_2 - y_1)^2 = x_2^2 + y_2^2 + x_1^2 + y_1^2 - 2(OP).(OQ)\cos\alpha$$

$$\Rightarrow \quad x_2^2 + x_1^2 - 2x_2x_1 + y_2^2 + y_1^2 - 2y_2y_1 = x_2^2 + y_2^2 + x_1^2 + y_1^2 - 2\,OP.OQ\cos\alpha$$

$$\Rightarrow \quad -2x_1x_2 - 2y_1y_2 = -2OP.OQ\cos\alpha$$

$$\Rightarrow \quad x_1x_2 + y_1y_2 = OP.\ OQ\cos\alpha$$

$$\Rightarrow \quad OP.\,OQ\cos\alpha = x_1x_2 + y_1y_2$$

Hence, proved.

Brief Review of Cartesian System of Rectangular co-ordinates Ex 22.1 Q2

We know that

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac},$$

where $a = BC, b = CA$ and $C = AB$ are the sides of the triangle ABC.

we have,

$$a = BC = \sqrt{(9-2)^2 + (2+1)^2} = \sqrt{49+9} = \sqrt{58}$$

$$b = CA = \sqrt{(0-9)^2 + (0+2)^2} = \sqrt{81+4} = \sqrt{85}$$

and, $$c = AB = \sqrt{(2-0)^2 + (-1-0)^2} = \sqrt{4+1} = \sqrt{5}$$

$$\therefore \quad \cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$= \frac{58 + 5 - 85}{2 \times \sqrt{58} \times \sqrt{5}}$$

$$= \frac{63 - 85}{2\sqrt{290}}$$

$$= \frac{-22}{2\sqrt{290}} = \frac{-11}{\sqrt{290}}$$

Hence, $\cos B = \dfrac{-11}{\sqrt{290}}$.

Brief Review of Cartesian System of Rectangular co-ordinates Ex 22.1 Q3

$A(6,3), B(-3,5), C(4,-2), D(x,3x)$

$$\text{or } (\square DBC) = \frac{1}{2}\left[x_1(y_2 - y_1) + x_2(y_3 - y_2) + x_3(y_1 - y_2)\right]$$

$$= \frac{1}{2}\left[-3(-2 - 3x) + 4(3x - 5) + x(5 + 2)\right]$$

$$= \frac{1}{2}\left[6 + 9x + 12x - 20 + 5x + 2x\right]$$

$$= \frac{1}{2}\left[28x - 14\right]$$

$$= 7\left[2x - 1\right]$$

$$\text{or } (\square ABC) = \frac{1}{2}\left[6(5+2) - 3(-2-3) + 4(3-5)\right]$$

$$= \frac{1}{2}\left[42 + 15 - 8\right]$$

$$= \frac{49}{2}$$

$$\frac{\text{or } (\square DBC)}{\text{or } (\square ABC)} = \frac{1}{2}$$

$$\frac{7(2x-1)}{\frac{49}{2}} = \frac{1}{2}$$

$$\frac{14(2x-1)}{49} = \frac{1}{2}$$

$$\frac{28x - 14}{49} = \frac{1}{2}$$

$$56x - 28 = 49$$

$$56x = 28 + 49$$

$$56x = 77$$

$$x = \frac{11}{8}$$

Brief Review of Cartesian System of Rectangular co-ordinates Ex 22.1 Q4

It is given that $A(2,0), B(9,1), C(11,6)$ and $D(4,4)$ are the vertices of a quadrilateral.

Now,

Coordinates of the mid-point of AC are $\left(\frac{2+11}{2}, \frac{0+6}{2}\right) = \left(\frac{13}{2}, 3\right)$

Coordinates of the mid-point of BD are $\left(\frac{9+4}{2}, \frac{1+4}{2}\right) = \left(\frac{13}{2}, \frac{5}{2}\right)$

Thus, AC and BD do not have the same mid-point. Hence $ABCD$ is not a paralleogram.

$\therefore$ $ABCD$ is not a rhombus.

Brief Review of Cartesian System of Rectangular co-ordinates Ex 22.1 Q5

Let $A(-36,7)$, $B(20,7)$ and $C(0,-8)$ be the vertices of the triangle ABC.

Now,

$$a = BC = \sqrt{(0-20)^2 + (-8-7)^2}$$

$$= \sqrt{400+225}$$

$$= \sqrt{625}$$

$$= 25,$$

$$b = AC = \sqrt{(0+36)^2 + (-8-7)^2}$$

$$= \sqrt{1296+225}$$

$$= \sqrt{1521}$$

$$= 39$$

and, $$c = AB = \sqrt{(20+36)^2 + (7-7)^2}$$

$$= \sqrt{(56)^2}$$

$$= 56$$

The coordinates of the centre of the circle are

$$\left(\frac{ax_1+bx_2+cx_3}{a+b+c}, \frac{ay_1+by_2+cy_3}{a+b+c}\right)$$

or, $$\left[\frac{25\times(-36)+39\times20+56\times0}{25+39+56}, \frac{25\times7+39\times7+56\times(-8)}{25+39+56}\right]$$

or, $$\left[\frac{-900+780}{120}, \frac{175+273-448}{120}\right]$$

or, $$\left[\frac{-120}{120}, \frac{0}{120}\right]$$

or, $(-1,0)$.

Hence, the coordinates of the centre of the circle are $(-1,0)$.

Brief Review of Cartesian System of Rectangular co-ordinates Ex 22.1 Q6

It is given that ABC is an equilatral triangle.

$\therefore \quad AB = BC = AC = 2a$

Area of equilatral triangle

$$= \frac{\sqrt{3}}{4}(\text{side})^2$$

$$= \frac{\sqrt{3}}{4} \times (2a)^2$$

$$= \frac{\sqrt{3}}{4} \times 4 \times a^2$$

$$= \sqrt{3}\,a^2$$

But, area of triangle $= \frac{1}{2} \times \text{Base} \times \text{Height}.$

$\Rightarrow \quad \frac{1}{2} \times \text{Base} \times \text{Height} = \sqrt{3}\,a^2$

$\Rightarrow \quad \frac{1}{2} \times BC \times OA = \sqrt{3}\,a^2$

$\Rightarrow \quad \frac{1}{2} \times 2a \times OA = \sqrt{3}\,a^2$

$\Rightarrow \quad OA = \sqrt{3}\,a$

$\therefore$ Coordinates of A are $\left(\sqrt{3}\,a, 0\right)$ or $OA \left(-\sqrt{3}\,a, 0\right)$

Clearly, the coordinates of B and C are $(0, -a)$ and $(0, a)$ respectively.

Hence, the vertices of the triangle are $(0, a), (0, -a)$ and $\left(-\sqrt{3}\,a, 0\right)$ or $(0, a), (0, -a)$ and $\left(\sqrt{3}\,a, 0\right)$.

Brief Review of Cartesian System of Rectangular co-ordinates Ex 22.1 Q7

It is given that $P(x_1, y_1)$ and $Q(x_2, y_2)$ are two points

(i) PQ is parallel to the y-axis.

$\therefore \quad x_1 = x_2$ ....(1)

$$\therefore \quad PQ = \left|\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\right|$$

$$= \left|\sqrt{(x_2 - x_x)^2 + (y_2 - y_1)^2}\right| \quad \text{[Using equation 1]}$$

$$= \left|\sqrt{(y_2 - y_1)^2}\right|$$

$$= |y_2 - y_1|$$

(ii) PQ is parallel to the x-axis.

$\therefore \quad y_1 = y_2$ ....(2)

$$\therefore \quad PQ = \left|\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\right|$$

$$= \left|\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\right| \quad \text{[Using equation 2]}$$

$$= \left|\sqrt{(x_2 - x_1)^2}\right|$$

$$= \left|\sqrt{x_2 - x_1}\right|$$

$$\therefore \quad PQ = |x_2 - x_1|$$

Brief Review of Cartesian System of Rectangular co-ordinates Ex 22.1 Q8

It is given that C lie on the x-axis. *Let* coordinates of C be $(x, 0)$.

Now, C *is* equidistant from the points $A(7, 6)$ and $B(3, 4)$.

$\therefore \quad AC = BC \quad$ [given]

$\Rightarrow \quad AC^2 = BC^2$

$$\Rightarrow \quad \left[\sqrt{(x-7)^2 + (0-6)^2}\right]^2 = \left[\sqrt{(x-3)^2 + (0-4)^2}\right]^2$$

$$\Rightarrow \quad (x-7)^2 + (-6)^2 = (x-3)^2 + (-4)^2$$

$$\Rightarrow \quad x^2 + 49 - 14x + 36 = x^2 + 9 - 6x + 16$$

$$\Rightarrow \quad 49 + 36 - 36 - 16 - 9 = x^2 - x^2 - 6x + 14x$$

$\Rightarrow \quad 85 - 25 = 8x$

$\Rightarrow \quad 60 = 8x$

$\Rightarrow \quad 8x = 60$

$$\Rightarrow \quad x = \frac{60}{8} = \frac{15}{2}$$

Hence, coordinates of c are $\left(\frac{15}{2}, 0\right)$.