RD Sharma Solutions Class 11 Maths Chapter 21 Ex 21.2

Some Special Series Ex 21.2 Q1

we have,

$$3+5+9+15+23+\ldots+T_{n-1}+T_n$$

The difference between the successive terms are $5-3=2$, $9-5=4$, $15-9=6$...clearly, these difference are in A.P.

Let, S_n denote the sum to n terms of the given series.

Then,

$$S_n = 3+5+9+15+23+\ldots\ldots T_{n-1}+T_n\ldots\text{(i)}$$

Also, $S_n = 3+5+9+15\ldots\ldots T_{n-1}+T_n\ldots\text{(ii)}$

Subtrating (ii) from (i), we get

$$0 = 3+\left[2+4+6+8\ldots\left(T_n-T_{n-1}\right)\right]-T_n$$

$$T_n = 3+\frac{(n-1)}{2}\left[2\times2+(n-1-1)\times2\right]$$

$$T_n = 3+\frac{(n-1)}{2}\times2\left[2+n-2\right]$$

$$= 3+(n-1)(n)$$

$$= 3+n^2-n$$

$$= n^2-n+3$$

$$\therefore \quad S_n = \sum_{k=1}^{n} T_k = \sum_{k=1}^{n}\left(k^2-k+3\right)$$

$$= \sum_{k=1}^{n} k^2 - \sum_{k=1}^{n} k + \sum_{k=1}^{n} 3$$

$$= \sum_{k=1}^{n} k^2 - \sum_{k=1}^{n} k + \sum_{k=1}^{n} 3$$

$$= \frac{n(n+1)(2n+1)}{6}\ \frac{-n(n+1)}{2}+3n$$

$$= \frac{n(n+1)(2n+1)-n(n+1)+18n}{6}$$

$$= \frac{n}{6}\left[(n+1)(2n+1)-3(n+1)+18\right]$$

$$= \frac{n}{6}\left[2n^2+n+2n+1-3n-3+18\right]$$

$$= \frac{n}{6}\left[2n^2+3n-3n-2+18\right]$$

$$= \frac{n}{6}\left[2n^2+16\right]$$

$$= \frac{n}{6}\times2\left[n^2+8\right]$$

$$= \frac{n}{3}\left(n^2+8\right)$$

Hence, $S_n = \frac{n}{3}\left(n^2+8\right)$

Some Special Series Ex 21.2 Q2

We have,

$2+5+10+17+26+....$

The sequence of the differences between the successive terms of this series is 3, 5, 7, 9.....
Cleary, it is an A.P. with common difference 2.
Let T_n be the nth term and S_n denote the sum of n terms of the given series.

Then, $S_n = 2+5+10+17...T_{n-1}+T_n...(i)$

Also, $S_n = 2+5+10+17....+T_{n-1}+T_n.....(ii)$

Subtracting (ii) from (i), we get.

$$0 = 2+\left[3+5+7...\left(T_n - T_{n-1}\right)\right] - T_n$$

$$\Rightarrow \quad T_n = 2+\left(3+5+7+....T_n - T_{n-1}\right)$$

$$= 2+\frac{(n-1)}{2}\left[2\times 3+(n-1-1)\times 2\right]$$

$$= 2+\frac{(n-1)}{2}\times 2\left[3+n-2\right]$$

$$= 2+(n-1)(n+1)$$

$$= 2+n^2+n-n-1$$

$$= 2+n^2-1$$

$$= n^2+1$$

$$\therefore \quad S_n = \sum_{k=1}^{n} T_k = \sum_{k=1}^{n}\left(k^2+1\right)$$

$$= \sum_{k=1}^{n} k^2 + \sum_{k=1}^{n} 1$$

$$\frac{n(n+1)(2n+1)}{6}+n$$

$$= \frac{n(n+1)(2n+1)+6n}{6}$$

$$= \frac{n}{6}\left[(n+1)(2n+1)+6\right]$$

$$= \frac{n}{6}\left[2n^2+n2n+1+6\right]$$

$$= \frac{n}{6}\left[2n^2+3n+7\right]$$

hence, $S_n = \frac{n}{6}\left[2n^2+3n+7\right]$

Some Special Series Ex 21.2 Q3

We have,

$1+3+7+13+21+\ldots\ldots.$

The sequence of the differences between the successive terms of this series is 2, 4, 6, 8....
clearly, it is an A.P. with common difference 2.
Let T_n be the nth term and S_n denote the sum of n terms of the given series.

Then, $S_n = 1+3+7+13+21+\ldots T_{n-1}+T_n \ldots\ldots(i)$

Also, $S_n = 1+3+7+13\ldots\ldots T_{n-1}+T_n \ldots.(ii)$

Subtracting (ii) from (i), we get

$$0 = 1+\left[2+4+6+8\ldots\ldots\left(T_n - T_{n-1}\right)\right]-T_n$$

$$\Rightarrow \quad T_n = 1+\left[2+4+6+8\ldots.\left(T_n - T_{n-1}\right)\right]$$

$$\Rightarrow \quad T_n = 1+\frac{(n-1)}{2}\left[2\times 2+(n-1-1)\times 2\right]$$

$$= 1+\frac{(n-1)}{2}\times 2\left[2+(n-2)\right]$$

$$= 1+(n-1)(n)$$

$$= 1+n^2-n$$

$$= n^2-n+1$$

$$\Rightarrow \quad S_n = \sum_{k=1}^{n} T_k = \sum_{k=1}^{n}\left(k^2-k+1\right) = \sum_{k=1}^{n} k^2 - \sum_{k=1}^{n} k + \sum_{k=1}^{n} 1$$

$$\Rightarrow \quad S_n = \sum_{k=1}^{n} k^2 - \sum_{k=1}^{n} k + \sum_{k=1}^{n} 1$$

$$= \frac{n(n+1)(2n+1)}{6} - \frac{n(n+1)}{2} + n$$

$$= \frac{n(n+1)(2n+1) - 3n(n+1) + 6n}{6}$$

$$\frac{n}{6}\left[(n+1)(2n+1) - 3(n+1) + 6\right]$$

$$= \frac{n}{6}\left[2n^2 + n + 2n + 1 - 3n - 3 + 6\right]$$

$$= \frac{n}{6}\left[2n^2 + 4\right]$$

$$= \frac{n}{6} \times 2\left[n^2 + 2\right]$$

$$= \frac{n}{3}\left(n^2 + 2\right)$$

Hence, $S_n = \frac{n}{3}\left(n^2 + 2\right)$.

Some Special Series Ex 21.2 Q4

We have,

$3 + 7 + 14 + 24 + 37 + \ldots\ldots\ldots$

The sequence of the differences between the successive terms of this series is $4, 7, 10, 13 + \ldots.$ clearly, it is an A.P. with common difference 3.

Let T_n be the nth term and S_n denote the sum of n terms of the given series.

Then, $S_n = 3 + 7 + 14 + 24 + 37 + \ldots T_{n-1} + T_n \ldots\ldots (i)$

Also, $S_n = 3 + 7 + 14 + 24 + \ldots\ldots + T_{n-1} + T_n \ldots. (ii)$

Subtracting (ii) from (i), we get

$$0 = 3 + \left[4 + 7 + 10\ldots\ldots + \left(T_n - T_{n-1}\right)\right] - T_n$$

$$\Rightarrow \quad T_n = 3 + \left[4 + 7 + 10\ldots. + \left(T_n - T_{n-1}\right)\right]$$

$$\Rightarrow \quad T_n = 3 + \frac{(n-1)}{2}\left[2 \times 4 + (n - 1 - 1) \times 3\right]$$

$$= 3 + \frac{(n-1)}{2}\left[8 + (n-2)3\right]$$

$$= 3 + \frac{(n-1)}{2}\left[8 + 3n - 6\right]$$

$$= 3 + \frac{(n-1)}{2}\left[2 + 3n\right]$$

$$= \frac{6 + (n-1)(2+3n)}{2}$$

$$= \frac{6 + 2n + 3n^2 - 2 - 3n}{2}$$

$$= \frac{6 + 3n^2 - n - 2}{2}$$

$$= \frac{3n^2 - n + 4}{2}$$

$$\Rightarrow \quad S_n = \sum_{k=1}^{n} T_k = \sum_{k=1}^{n} \frac{\left(3k^2 - k + 4\right)}{2} = \frac{1}{2}\left[\sum_{k=1}^{n} 3k^2 - \sum_{k=1}^{n} k + \sum_{k=1}^{n} 4\right]$$

$$= \frac{3}{2}\sum_{k=1}^{n} k^2 - \frac{1}{2}\sum_{k=1}^{n} k + \sum_{k=1}^{n} 2$$

$$= \frac{3}{2}\left[\frac{n(n+1)(2n+1)}{6}\right] - \frac{1}{2}\left[\frac{n(n+1)}{2}\right] + 2n$$

$$= \frac{n(n+1)(2n+1)}{4} - \frac{n(n+1)}{4} + 2n$$

$$= \frac{n(n = 1)(2n+1) - n(n+1) + 8n}{4}$$

$$= \frac{n}{4}\left[(n+1)(2n+1) - (n+1) + 8\right]$$

$$= \frac{n}{4}\left[2n^2 + n + 2n + 1 - n - 1 + 8\right]$$

$$= \frac{n}{4}\left[2n^2 + 2n + 8\right]$$

$$= \frac{n}{4} \times 2\left[n^2 + n + 4\right]$$

$$= \frac{n}{2}\left[n^2 + n + 4\right]$$

Hence, $S_n = \frac{n}{2}\left[n^2 + n + 4\right]$

Some Special Series Ex 21.2 Q5

We have,

$$1 + 3 + 6 + 10 + 15 + \ldots\ldots.$$

The sequence of the differences between the successive terms of this series is 2, 3, 4, 5 + ...
clearly, it is an A.P. with common difference 1.
Let T_n be the nth term and S_n denote the sum of n terms of the given series.

Then, $S_n = 1 + 3 + 6 + 10 + 15 + \ldots T_{n-1} + T_n \ldots\ldots (i)$
Also, $S_n = 1 + 3 + 6 + 10 + \ldots.. + T_{n-1} + T_n \ldots. (ii)$

Subtracting (ii) from (i), we get

$$0 = 1 + \left[2 + 3 + 4 + 5 \ldots..\left(T_n - T_{n-1}\right)\right] - T_n$$

$$\Rightarrow \quad T_n = 1 + \left[2 + 3 + 4 + 5 \ldots.\left(T_n - T_{n-1}\right)\right]$$

$$\Rightarrow \quad T_n = 1 + \frac{(n-1)}{2}\left[2 \times 2 + (n - 1 - 1) \times 1\right]$$

$$= 1 + \frac{(n-1)}{2}[4 + n - 2]$$

$$= 1 + \frac{(n-1)}{2}(n + 2)$$

$$= 1 + \frac{n^2 + 2n - n - 2}{2}$$

$$= 1 + \frac{n^2 + n - 2}{2}$$

$$= \frac{2 + n^2 + n - 2}{2}$$

$$= \frac{n^2 + n}{2}$$

$$\therefore \quad = \sum_{k=1}^{n} T_k = \sum_{k=1}^{n}\left(\frac{k^2 + k}{2}\right) = \frac{1}{2}\sum_{k=1}^{n} k^2 + \frac{1}{2}\sum_{k=1}^{n} k$$

$$\Rightarrow \qquad S_n = \frac{1}{2}\sum_{k=1}^{n} k^2 + \frac{1}{2}\sum_{k=1}^{n} k$$

$$= \frac{1}{2}\left[\frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}\right]$$

$$= \frac{1}{2}\left[n(n+1)\left[\frac{2n+1}{6} + \frac{1}{2}\right]\right]$$

$$= \frac{n(n+1)}{2}\left[\frac{2n+1+3}{6}\right]$$

$$= \frac{n(n+1)}{2}\left[\frac{2n+4}{6}\right]$$

$$= \frac{n(n+1)\times 2[n+2]}{2\times 6}$$

$$= \frac{n}{6}(n+1)(n+2)$$

Hence, $S_n = \frac{n}{6}(n+1)(n+2)$.

Some Special Series Ex 21.2 Q6

We have,

$$1+4+13+40+121+.......$$

The sequence of the differences between the successive terms of this series is 3, 9, 27, 81....
clearly, it is a G.P. with common difference 3.
Let T_n be the nth term and S_n denote the sum of n terms of the given series.

Then, $S_n = 1+4+13+40+......+T_{n-1}+T_n......(i)$

Also, $S_n = 1+4+13.....+T_{n-1}+T_n....(ii)$

Subtracting (ii) from (i), we get

$$0 = 1+\left[3+9+27+81.....\left(T_n - T_{n-1}\right)\right]-T_n$$

$$\Rightarrow \quad T_n = 1+\left[3+9+27+81....\left(T_n - T_{n-1}\right)\right]$$

$$\Rightarrow \quad T_n = 1+\frac{3\left(3^{n-1}-1\right)}{\left(3-1\right)}$$

$$\Rightarrow \quad T_n = 1+\frac{3}{2}\left(3^{n-1}-1\right)$$

$$= 1+\frac{3}{2}-3^{n-1}-\frac{3}{2}$$

$$= 1-\frac{3}{2}+\frac{3^n}{2}$$

$$= -\frac{1}{2}+\frac{3^n}{2}$$

$$= \frac{3^n}{2}-\frac{1}{2}$$

$$\therefore \quad S_n = \sum_{k=1}^{n} T_k = \sum_{k=1}^{n}\left(\frac{3^n}{n}-\frac{1}{2}\right)$$

$$= \sum_{k=1}^{n} \frac{3^n}{n} - \frac{1}{2} \sum_{k=1}^{n} 1$$

$$= \frac{1}{2} \sum_{k=1}^{n} 3^n - \frac{1}{2} \sum_{k=1}^{n} 1$$

$$\Rightarrow \quad S_n = \frac{1}{2}\left[3^1 + 3^2 + 3^3 + \ldots 3^n\right] - \frac{1}{2} \times n$$

$$= \frac{1}{2}\left[3 \times \frac{\left(3^n - 1\right)}{3-1}\right] - \frac{n}{2}$$

$$= \frac{3\left(3^{n-1}\right)}{4} - \frac{n}{2}$$

$$= \frac{3.3^n - 3 - 2n}{4}$$

$$= \frac{3^{n+1} - 2n - 3}{4}$$

Hence, $S_n = \dfrac{3^{n+1} - 2n - 3}{4}$

Some Special Series Ex 21.2 Q7

We have,

$$4+6+9+13+18+\ldots\ldots.$$

The sequence of the differences between the successive terms of this series is 2, 3, 4, 5....
clearly, it is an A.P. with common difference 1.
Let T_n be the nth term and S_n denote the sum of n terms of the given series.

Then, $S_n = 4+6+13+18\ldots\ldots T_{n-1}+T_n\ldots\ldots(i)$
Also, $S_n = 4+6+9+13\ldots\ldots T_{n-1}+T_n\ldots.(ii)$

Subtracting (ii) from (i), we get

$$0 = 4+\left[2+3+4+5\ldots..\left(T_n - T_{n-1}\right)\right]-T_n$$

$$\Rightarrow \quad T_n = 4+\left[2+3+4+5\ldots.\left(T_n - T_{n-1}\right)\right]$$

$$\Rightarrow \quad T_n = 4+\frac{(n-1)}{2}\left[2\times2+(n-1-1)\times1\right]$$

$$= 4+\frac{(n-1)}{2}\left[4+n-2\right]$$

$$= 4+\frac{(n-1)}{2}(n+2)$$

$$= \frac{8+n^2+2n-n-2}{2}$$

$$= \frac{n^2+n+6}{2}$$

$$\therefore \quad S_n = \sum_{k=1}^{n} T_k = \sum_{k=1}^{n}\left(\frac{k^2+k+6}{2}\right)$$

$$= \frac{1}{2}\sum_{k=1}^{n} k^2 + \frac{1}{2}\sum_{k=1}^{n} k + \sum_{k=1}^{n} 3$$

$$\Rightarrow \quad S_n = \frac{1}{2}\left[\frac{n(n+1)(2n+1)}{6}\right]+\frac{n(n+1)}{2\times2}+3n$$

$$\Rightarrow \quad S_n = \frac{1}{12}(n)(n+1)(2n+1) + \frac{n(n+1)}{4} + 3n$$

$$= \frac{n(n+1)(2n+1) + 3n(n+1) + 36n}{12}$$

$$= n\left[\frac{(n+1)(2n+1) + 3(n+1) + 36}{12}\right]$$

$$= \frac{n}{12}\left[2n^2 + n + 2n + 1 + 3n + 3 + 36\right]$$

$$= \frac{n}{12}\left[2n^2 + 6n + 40\right]$$

$$= \frac{2n}{12}\left[n^2 + 3n + 20\right]$$

$$= \frac{n}{6}\left(n^2 + 3n + 20\right)$$

Hence, $S_n = \frac{n}{6}\left(n^2 + 3n + 20\right)$

Some Special Series Ex 21.2 Q8

We have,

$$2 + 4 + 7 + 11 + 16 + \ldots\ldots.$$

The sequence of the differences between the successive terms of this series is 2, 3, 4, 5....
clearly, it is an A.P. with common difference 1.
Let T_n be the nth term and S_n denote the sum of n terms of the given series.

Then, $S_n = 2 + 4 + 7 + 11 + 16 \ldots\ldots + T_{n-1} + T_n \ldots\ldots (i)$

Also, $S_n = 2 + 4 + 7 + 11 + \ldots\ldots T_{n-1} + T_n \ldots\ldots (ii)$

Subtracting (ii) from (i), we get

$$0 = 2 + \left[2 + 3 + 4 + 5 \ldots\ldots (T_n - T_{n-1})\right] - T_n$$

$$\Rightarrow \quad T_n = 2 + \frac{(n-1)}{2}\left[2 \times 2 + (n - 1 - 1) \times 1\right]$$

$$= 2 + \frac{(n-1)}{2}\left[4 + n - 2\right]$$

$$= 2 + \frac{(n-1)}{2}(n + 2)$$

$$= \frac{4 + n^2 + 2n - n - 2}{2}$$

$$= \frac{n^2 + n + 2}{2}$$

$$= \frac{n^2}{2} + \frac{n}{2} + 1$$

$$\Rightarrow \quad S_n = \sum_{k=1}^{n} T_k = \sum_{k=1}^{n}\left(\frac{k^2}{2} + \frac{k}{2} + 1\right)$$

$$= \frac{1}{2}\sum_{k=1}^{n} n^2 + \frac{1}{2}\sum_{k=1}^{n} n + \sum_{k=1}^{n} 1$$

$$= \frac{1}{2}\left[\frac{n(n+1)(2n+1)}{6}\right] + \frac{1}{2}\left[\frac{n(n+1)}{2}\right] + n$$

$$= \frac{n(n+1)(2n+1)}{12} + \frac{n(n+1)}{4} + n$$

$$= \frac{n(n+1)(2n+1) + 3n(n+1) + 12n}{12}$$

$$= \frac{n}{12}\left[(n+1)(2n+1) + 3(n+1) + 12\right]$$

$$= \frac{n}{12}\left[2n^2 + n + 2n + 1 + 3n + 3 + 12\right]$$

$$= \frac{n}{12}\left[2n^2 + 6n + 16\right]$$

$$= \frac{2n}{12}\left[n^2 + 3n + 8\right]$$

$$= \frac{n}{6}\left[n^2 + 3n + 8\right]$$

Hence, $S_n = \frac{n}{6}\left(n^2 + 3n + 8\right)$

Some Special Series Ex 21.2 Q9

We have,

$$\frac{1}{1.4}+\frac{1}{4.7}+\frac{1}{7.10}+.....$$

Let T_r be the rth term of the given series. Then,

$$T_r = \frac{1}{(3r-2)(3r+1)}, r = 1, 2..., n$$

$$\Rightarrow \quad T_r = \frac{1}{3}\left[\frac{1}{3r-2} - \frac{1}{3r+1}\right]$$

$$\therefore \quad \text{required sum} = \frac{1}{3}\sum_{r=1}^{n} T_r$$

$$= \frac{1}{3}\sum_{r=1}^{n}\left[\frac{1}{3n-2} - \frac{1}{3n+1}\right]$$

$$= \frac{1}{3}\left[\left(1-\frac{1}{4}\right)+\left(\frac{1}{4}-\frac{1}{7}\right)+\left(\frac{1}{7}-\frac{1}{10}\right)....\left(\frac{1}{3n-2}-\frac{1}{3n+1}\right)\right]$$

$$= \frac{1}{3}\left[1-\frac{1}{3n+1}\right]$$

$$= \frac{1}{3}\left[\frac{3n+1-1}{3n+1}\right]$$

$$= \frac{1}{3}\times\frac{3n}{3n+1}$$

$$= \frac{n}{3n+1}$$

Hence, required sum $= \dfrac{n}{3n+1}$

Some Special Series Ex 21.2 Q10

We have,

$$\frac{1}{1.6}+\frac{1}{6.11}+\frac{1}{11.14}+\frac{1}{14.19}+\ldots..\frac{1}{(5n-4)(5n+1)}$$

Let T_r be the rth term of the given series. Then,

$$T_r = \frac{1}{(5r-4)(5r+1)}, r = 1, 2 \ldots, n$$

$$\Rightarrow \qquad T_r = \frac{1}{5}\left[\frac{1}{5r-2}-\frac{1}{5r+1}\right]$$

$$\therefore \quad \text{required sum} = \frac{1}{5}\sum_{r=1}^{n} T_r$$

$$= \frac{1}{5}\sum_{r=1}^{n}\left[\frac{1}{5r-4}-\frac{1}{5r+1}\right]$$

$$= \frac{1}{5}\left[\left(1-\frac{1}{6}\right)+\left(\frac{1}{6}-\frac{1}{11}\right)+\left(\frac{1}{11}-\frac{1}{14}\right)\ldots.\left(\frac{1}{5n-4}-\frac{1}{5n+1}\right)\right]$$

$$= \frac{1}{5}\left[1-\frac{1}{5n+1}\right]$$

$$= \frac{1}{5}\left[\frac{5n+1-1}{5n+1}\right]$$

$$= \frac{1}{5}\times\frac{5n}{5n+1}$$

$$= \frac{n}{5n+1}$$

Hence, required sum $= \dfrac{n}{5n+1}$