RD Sharma Solutions Class 11 Maths Chapter 20 Ex 20.4

Geometric Progressions Ex 20.4 Q 1

$$S_\infty = 1 - \frac{1}{3} + \frac{1}{3^2} - \frac{1}{3^3} + \ldots$$

$$\Rightarrow \quad a = 1,\ r = -\frac{1}{3}$$

$$S_\infty = \frac{a}{1-r}$$

$$= \frac{1}{1+\frac{1}{3}}$$

$$S_\infty = \frac{3}{4}$$

$$S_\infty = 8 + 4\sqrt{2} + 4 + \ldots$$

$$\Rightarrow \quad a = 8,\ r = \frac{4}{4\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$S_\infty = \frac{a}{1-r}$$

$$= \frac{8}{1-\frac{1}{\sqrt{2}}}$$

$$= \frac{8\sqrt{2}}{\sqrt{2}-1} \times \frac{\left(\sqrt{2}+1\right)}{\left(\sqrt{2}+1\right)}$$

$$= \frac{8\left(2+\sqrt{2}\right)}{2-1}$$

$$S_\infty = 8\left(2+\sqrt{2}\right)$$

$$S_\infty = \frac{2}{5} + \frac{3}{5^2} + \frac{2}{5^3} + \frac{3}{5^4} + \dots$$
$$= \left(\frac{2}{5} + \frac{2}{5^3} + \dots\right) + \left(\frac{3}{5^2} + \frac{3}{5^4} + \dots\right)$$
$$S_\infty = S'_\infty + S''_\infty$$

For

$$S'_\infty = \frac{a}{1-r}$$
$$= \frac{\frac{2}{5}}{1 - \frac{1}{25}}$$
$$= \frac{2}{5} \times \frac{25}{24}$$
$$S'_\infty = \frac{5}{12}$$
$$S''_\infty = \frac{\frac{3}{25}}{1 - \frac{1}{25}}$$
$$= \frac{3}{25} \times \frac{25}{24}$$
$$= \frac{3}{24}$$
$$S_\infty = S'_\infty + S''_\infty$$
$$= \frac{5}{12} + \frac{3}{24}$$
$$= \frac{13}{24}$$
$$S_\infty = \frac{13}{24}$$

This infinite G.P has first term $a = 10$ and common ratio $r = -\frac{9}{10} = -0.9$

Thus the sum of the infinite G.P will be:

$$10 - 9 + 8.9 - 7.29 + \cdots \infty = \frac{a}{1-r} \quad \left[\text{Since } |r| < 1\right]$$
$$= \frac{10}{1-(-0.9)}$$
$$= \frac{10}{1.9}$$
$$= \frac{100}{19}$$

The G.P can be written as follows:

$$\frac{1}{3}+\frac{1}{5^2}+\frac{1}{3^3}+\frac{1}{5^4}+\frac{1}{3^5}+\frac{1}{5^6}+\cdots\infty=\left(\frac{1}{3}+\frac{1}{3^3}+\frac{1}{3^5}+\cdots\infty\right)+\left(\frac{1}{5^2}+\frac{1}{5^4}+\frac{1}{5^6}+\cdots\infty\right)$$

$$=\frac{\frac{1}{3}}{1-\frac{1}{3^2}}+\frac{\frac{1}{5^2}}{1-\frac{1}{5^2}}$$

$$=\frac{3}{8}+\frac{1}{24}$$

$$=\frac{10}{24}$$

$$=\frac{5}{12}$$

Geometric Progressions Ex 20.4 Q 2

$$9^{\frac{1}{3}}\times 9^{\frac{1}{9}}\times 9^{\frac{1}{27}}\ldots\infty$$

$$=9^{\left(\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+000\right)}$$

$$=9^{\left(\frac{\frac{1}{3}}{1-\frac{1}{3}}\right)} \qquad \left[\text{Using } S_\infty=\frac{a}{1-r}\right]$$

$$=9^{\left(\frac{1}{3}\times\frac{3}{2}\right)}$$

$$=9^{\frac{1}{2}}$$

$$=3$$

So,

$$9^{\frac{1}{3}}\times 9^{\frac{1}{9}}\times 9^{\frac{1}{27}}\ldots\infty=3$$

Geometric Progressions Ex 20.4 Q 3

$2^{\frac{1}{4}}.4^{\frac{1}{8}}.8^{\frac{1}{16}}.16^{\frac{1}{32}}......\infty$

$= 2^{\frac{1}{4}}.2^{\frac{2}{8}}.2^{\frac{3}{16}}.2^{\frac{4}{32}}.....\infty$

$= \left(\frac{1}{4}+\frac{2}{8}+\frac{3}{16}+\frac{4}{32}+.....\infty\right)$

$= 2$

$= 2^{5} ------(1)$

$S = \frac{1}{4}+\frac{2}{8}+\frac{3}{16}+\frac{4}{32}+.....\infty$

$S = \left(\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}+.....\infty\right)2$

$\frac{S}{2} = \frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}+.....\infty$

$= \frac{\frac{1}{4}}{1-\frac{1}{2}}$

$= \frac{1}{4}\times\frac{2}{1}$

$S = \frac{1}{2}$

$S = 1$

Thus $2^{\frac{1}{4}}.4^{\frac{1}{8}}.8^{\frac{1}{16}}.16^{\frac{1}{32}}...... = 2^{1} = 2$

Geometric Progressions Ex 20.4 Q4

$S_p = 1 + r^p + r^{2p} + ... + \infty$

$S_p = \frac{1}{1-r^p}$

$s_p = 1 - r^p + r^{2p} + ... + \infty$

$s_p = \frac{1}{1+r^p}$

Now,

$S_p + s_p = \frac{1}{1-r^p} + \frac{1}{1+r^p}$

$= \frac{2}{1-r^{2p}}$

$S_p + s_p = 2\times S_{2p}$

Geometric Progressions Ex 20.4 Q5.

Here,

$$a = 4$$

$$A_3 - a_5 = \frac{31}{81}$$

$$ar^2 - ar^4 = \frac{32}{81}$$

$$r^2 4\left(1 - r^2\right) = \frac{32}{81}$$

$$r^2\left(1 - r^2\right) = \frac{8}{81}$$

Let $r^2 = A$

$$A(1 - A) = \frac{8}{81}$$

$$A - A^2 = \frac{8}{81}$$

$$81A - 81A^2 = 8$$

$$81A^2 - 81A + 8 = 0$$

$$A = \frac{81 \pm \sqrt{(81)^2 - 4 \times 81 \times 8}}{81 \times 2}$$

$$= \frac{81 \pm \sqrt{6561 - 2592}}{162}$$

$$= \frac{81 \pm \sqrt{3969}}{162}$$

$$= \frac{81 \pm 63}{162}$$

$$= \frac{81 + 63}{162} \text{ or } \frac{81 - 63}{162}$$

$$= \frac{144}{162} \text{ or } \frac{18}{162}$$

$$r^2 = \frac{8}{9} \text{ or } \frac{1}{9}$$

$$r = \pm\frac{2\sqrt{2}}{3} \text{ or } \pm\frac{1}{3}$$

Since it is a decreasing G.P.

$$r = \frac{2\sqrt{2}}{3}, \frac{1}{3}$$

$$S_\infty = \frac{4}{1 - \frac{2\sqrt{2}}{3}} \text{ and } S_\infty = \frac{4}{1 - \frac{1}{3}}$$

$$S_\infty = \frac{12}{3 - 2\sqrt{2}}, 6$$

Geometric Progressions Ex 20.4 Q6.

$a = 1$

$$a_n = a_{n+1} + a_{n+2} + a_{n+3} + \dots\dots\dots$$

$$ar^{n-1} = ar^n + ar^{n+1} + ar^{n+2} + \dots\dots\dots$$

$$ar^{n-1} = ar^n\left(1 + r + r^2 + \dots\dots\infty\right)$$

$$1 = r\left(\frac{1}{1 - r}\right)$$

$$1 - r = r$$

$$1 = 2r$$

$$r = \frac{1}{2}$$

G.P. is $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \dots\dots\dots$

Geometric Progressions Ex 20.4 Q 7

$$a + ar = 5$$
$$a(1 + r) = 5 \text{-----} (1)$$
$$a_n = 3(a_{n+1} + a_{n+2} + a_{n+3} + \ldots\ldots)$$
$$ar^{n-1} = 3(ar^n + ar^{n+1} + ar^{n+2} + \ldots\ldots)$$
$$ar^{n-1} = 3ar^n(1 + r + r^2 + \ldots\ldots\infty)$$
$$1 = 3r\left(\frac{1}{1-r}\right)$$
$$1 - r = 3r$$
$$1 = 4r$$
$$r = \frac{1}{4}$$
$$a(1 + r) = 5$$
$$a\left(\frac{5}{4}\right) = 5$$
$$a = 4$$

G.P. is $4, 1, \frac{1}{4}, \frac{1}{16}, \ldots\ldots$

Geometric Progressions Ex 20.4 Q8

$$0.125125125\ldots\ldots = 0.\overline{125}$$
$$= 0.125 + 0.000125 + 0.000000125 + \ldots$$
$$= \frac{125}{10^3} + \frac{125}{10^6} + \frac{125}{10^9} + \ldots$$
$$= \frac{125}{10^3}\left(1 + \frac{1}{10^3} + \frac{1}{10^6} + \ldots\right)$$
$$= \frac{125}{10^3}\left(\frac{1}{1 - \frac{1}{1000}}\right)$$
$$= \frac{125}{1000}\left(\frac{1000}{999}\right)$$
$$0.125125125\ldots\ldots = \frac{125}{999}$$

Geometric Progressions Ex 20.4 Q 9

$$0.4\overline{23} = 0.4 + 0.0232323.........$$

$$= 0.4 + 0.023 + 0.00023 + ...$$

$$= 0.4 + \frac{23}{10^3} + \frac{23}{10^5} + ...$$

$$= 0.4 + \frac{23}{10^3}\left(1 + \frac{1}{10^2} + \frac{1}{10^4} + ...\right)$$

$$= 0.4 + \frac{23}{1000}\left(\frac{1}{1 - \frac{1}{100}}\right)$$

$$= 0.4 + \frac{23}{1000}\left(\frac{100}{99}\right)$$

$$= \frac{4}{10} + \frac{23}{990}$$

$$= \frac{396 + 23}{990}$$

$$0.4\overline{23} = \frac{419}{990}$$

Geometric Progressions Ex 20.4 Q 10

Let a be first term and r be common ratio of G.P. Here,

$$\frac{a_n}{(a_{n+1} + a_{n+2} + \ldots\infty)} = \frac{ar^{n-1}}{ar^n + ar^{n+1} + \ldots}$$

$$= \frac{ar^{n-1}}{ar^n\left(1 + r + r^2 + ..\infty\right)}$$

$$= \frac{ar^{n-1}}{ar^n\left(\frac{1}{1-r}\right)}$$

$$= \left(\frac{1-r}{r}\right)$$

Since r is a constant, so

$$\left(\frac{a_n}{a_{n+1} + a_{n+2} + \ldots\infty}\right) = k \text{ (constant)}$$

Such that $k = \left(\frac{1-r}{r}\right)$

Geometric Progressions Ex 20.4 Q 11

$$0.\overline{3} = 0.3333\ldots$$
$$= 0.3 + 0.03 + 0.003 + \ldots$$
$$= \frac{3}{10} + \frac{3}{10^2} + \frac{3}{10^3} + \ldots$$
$$= \frac{3}{10}\left(1 + \frac{1}{10} + \frac{1}{10^2} + \ldots\right)$$
$$= \frac{3}{10}\left(\frac{1}{1 - \frac{1}{10}}\right)$$
$$= \frac{3}{10} \times \frac{10}{9}$$
$$= \frac{3}{9}$$
$$0.\overline{3} = \frac{1}{3}$$

$$0.\overline{231} = 0.231231231\ldots$$
$$= 0.231 + 0.000231 + 0.000000231$$
$$= \frac{231}{10^3} + \frac{231}{10^6} + \frac{231}{10^9} + \ldots$$
$$= \frac{231}{10^3}\left(1 + \frac{1}{10^3} + \frac{1}{10^6} + \ldots\right)$$
$$= \frac{231}{1000}\left(\frac{1}{1 - \frac{1}{1000}}\right)$$
$$0.\overline{231} = \frac{231}{999}$$

$$3.5\overline{2} = 3 + 0.52222\ldots$$
$$= 3 + 0.5 + 0.02 + 0.002 + 0.0002 + \ldots$$
$$= 3.5 + \frac{2}{10^2} + \frac{2}{10^3} + \frac{2}{10^4} + \ldots$$
$$= 3.5 + \frac{2}{10^2}\left(1 + \frac{1}{10} + \frac{1}{10^2} + \ldots\right)$$
$$= \frac{35}{10} + \frac{2}{100}\left(\frac{1}{1 - \frac{1}{10}}\right)$$
$$= \frac{35}{10} + \frac{2}{100} \times \left(\frac{10}{9}\right)$$
$$= \frac{35}{10} + \frac{2}{90}$$
$$= \frac{315 + 2}{90}$$
$$3.5\overline{2} = \frac{317}{90}$$

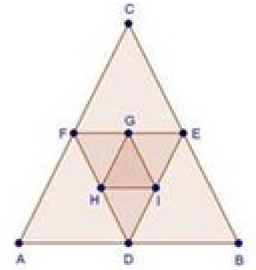
The rational number can be written as:

$$0.6\overline{8} = 0.6 + 0.08 + 0.008 + 0.0008 + \cdots\infty$$
$$= \frac{3}{5} + 8[0.01 + 0.001 + 0.0001 + \cdots\infty]$$
$$= \frac{3}{5} + 8\left[\frac{1}{100} + \frac{1}{1000} + \cdots\infty\right]$$

This is an infinite GP with first term $\frac{1}{100}$ and common ratio $\frac{1}{10}$

$$= \frac{3}{5} + 8 \cdot \frac{1}{100} \cdot \frac{1}{1 - \frac{1}{10}}$$
$$= \frac{3}{5} + \frac{4}{45}$$
$$= \frac{31}{45}$$

Geometric Progressions Ex 20.4 Q12



Side of triangle $= 18$ cm.

$AD = BD = 9$ cm.

$DE = BD = 9$ cm.

$GI = IF = \frac{9}{2}$ cm.

Sides of the triangles are $18, 9, \frac{9}{2}$.........

(i) sum of perimeters of the equilateral triangle $= \left(54 + 27 + \frac{27}{2} + ...\right)$

$$= \frac{54}{1 - \frac{1}{2}}$$

$$= 54 \times 2$$

Perimeter $= 108$ cm.

(ii) sum of area of equilateral triangle

$$= \left[\frac{\sqrt{3}}{4}(18)^2 + \frac{\sqrt{3}}{4}(9)^2 + \frac{\sqrt{3}}{4}\left(\frac{9}{2}\right)^2 + ...\right]$$

$$= \frac{\sqrt{3}}{4}\left[324 + 81 + \frac{81}{4} + ...\right]$$

$$= \frac{\sqrt{3}}{4}\left[\frac{324}{1 - \frac{1}{4}}\right]$$

$$= \frac{\sqrt{3}}{4}\left[\frac{324 \times 4}{3}\right]$$

$$= \sqrt{3}\,(108)$$

Geometric Progressions Ex 20.4 Q13

$$S = a + ar + ar^2 + ar^3 + \ldots\ldots$$

$$S = \frac{a}{1-r} \text{-----} (1)$$

$$S_1 = a^2 + a^2r^2 + a^2r^4 + a^2r^6 \ldots\ldots$$

$$S_1 = \frac{a^2}{1-r^2} \text{-----} (2)$$

$$S^2 = \frac{a^2}{(1-r)^2}$$

$$S^2 = \frac{S_1(1-r^2)}{(1-r^2)}$$

$$(1-r)S^2 = S_1(1+r)$$

$$S^2 - S^2r = S_1 + S_1r$$

$$S_1r + S^2r = S^2 - S_1$$

$$r = \frac{S^2 - S_1}{S_1 + S^2}$$

Put r in equation (1)

$$S(1-r) = a$$

$$a = S\left[1 - \frac{S^2 - S_1}{S^2 + S_1}\right]$$

$$a = S\left[\frac{S^2 + S_1 - S^2 + S_1}{S^2 + S_1}\right]$$

$$a = \frac{2SS_1}{S^2 + S_1}$$