RD Sharma Solutions Class 11 Maths Chapter 20 Ex 20.3

Geometric Progressions Ex 20.3 Q 1

2, 6, 18, ...to 7 term

$$a = 2,\ r = \frac{6}{2} = 3,\ n = 7$$

$$S_n = a\frac{\left(r^n - 1\right)}{r - 1}$$

$$S_7 = 2\frac{\left(3^7 - 1\right)}{3 - 1} = \frac{2}{2}\left(3^7 - 1\right)$$

$$= 2187 - 1 = 2186$$

1, 3, 9, 27, ... to 8 terms

$$a = 1,\ r - \frac{3}{1} = 3,\ n = 8$$

$$S_n = a\frac{\left(r^n - 1\right)}{r - 1}$$

$$S_8 = 1\frac{\left(3^8 - 1\right)}{3 - 1} = 3280$$

$1, \frac{-1}{2}, \frac{1}{4}, \frac{-1}{8}, \ldots, 9$ terms

$$a = 1,\ r = \frac{\frac{-1}{2}}{1} = \frac{-1}{2},\ n = 9$$

$$S_n = a\frac{\left(r^n - 1\right)}{r - 1}$$

$$S_9 = 1\frac{\left(\frac{-1}{2}\right)^9 - 1}{\frac{-1}{2} - 1}$$

$$= \frac{\frac{-1}{512} - 1}{\frac{-1}{2} - 1}$$

$$= \frac{\frac{-1 - 512}{512}}{\frac{-1 - 2}{2}}$$

$$= \frac{-513}{512} \times \frac{2}{-3}$$

$$= \frac{171}{256}$$

$\left(a^2 - b^2\right),\ (a - b),\ \left(\frac{a - b}{a + b}\right), \ldots n$ terms

$$a = a^2 - b^2,\ r = \frac{a - b}{a^2 - b^2} = \frac{1}{a + b},\ n = n$$

$$S_n = a\frac{\left(1 - r^n\right)}{1 - r} \qquad [\because r < 1]$$

$$S_n = \left(a^2 - b^2\right)\frac{\left(1 - \frac{1}{(a + b)^n}\right)}{1 - \frac{1}{a + b}}$$

$$= \frac{(a - b)\left((a + b)^n - 1\right)}{\frac{(a + b)^{-1}(a + b)^n\,(a + b) - 1}{(a + b)}}$$

$$= \frac{a - b}{(a + b)^n}\frac{\left((a + b)^n - 1\right)}{(a + b) - 1}$$

$4, 2, 1, \frac{1}{2}, \ldots 10$ terms

$$a = 4,\ r = \frac{2}{4} = \frac{1}{2},\ n = 10$$

$$S_n = a\frac{\left(1 - r^n\right)}{1 - r}$$

$$= 4\frac{1 - \left(\frac{1}{2}\right)^{10}}{1 - \frac{1}{2}}$$

$$= 8\left(1 - \frac{1}{2^{10}}\right)$$

$$= 8\left(1 - \frac{1}{1024}\right)$$

Geometric Progressions Ex 20.3 Q 2

$0.15 + 0.015 + 0.0015 + \ldots\ldots$ upto 8 terms

$= 15\left(0.1 + 0.01 + 0.001 + \ldots\ldots \text{upto 8 terms}\right)$

$$= 15\left(\frac{1}{10} + \frac{1}{100} + \ldots\ldots\right)$$

$$r = \frac{1}{10}, a = \frac{1}{10}$$

$$Sum = 15\left(\frac{\frac{1}{10}\left(1 - \frac{1}{10^8}\right)}{1 - \frac{1}{10}}\right)$$

$$= \frac{5}{3}\left(1 - \frac{1}{10^8}\right)$$

Here the first term of the series is $a = \sqrt{2}$ and the common ration is $r = \frac{\frac{1}{\sqrt{2}}}{\sqrt{2}} = \frac{1}{2}$

Thus the sum of the G.P up to 8^{th} terms is:

$$S_8 = \frac{a\left(1 - r^8\right)}{1 - r} = \frac{\sqrt{2}\left(1 - \left(\frac{1}{2}\right)^8\right)}{1 - \frac{1}{2}} = 2\sqrt{2}\left(1 - \frac{1}{256}\right) = \frac{255\sqrt{2}}{128}$$

$\frac{2}{9}-\frac{1}{3}+\frac{1}{2}-\frac{3}{4}+\ldots$ to 5 terms.

$$a=\frac{2}{9},\ r=\frac{\frac{-1}{3}}{\frac{2}{9}}=\frac{-1}{3}\times\frac{9}{2}=\frac{-3}{2},\ n=5$$

$$S_5=a\frac{\left(1-r^5\right)}{1-r}$$

$$=\frac{2}{9}\frac{\left(1-\left(\frac{-3}{2}\right)^5\right)}{1-\left(\frac{-3}{2}\right)}$$

$$=\frac{2}{9}\frac{\left(1+\frac{243}{32}\right)}{1+\frac{3}{2}}$$

$$=\frac{2}{9}\frac{(275)}{32}\times\frac{2}{5}$$

$$=\frac{55}{72}$$

$$(x+y)+(x^2+xy+y^2)+(x^3+x^2y+xy^2+y^3)+\ldots\ldots$$

$$=\frac{1}{x-y}\{(x^2-y^2)+(x^3-y^3)+\ldots\ldots..to\ \infty\}\ldots\ldots.[\because\frac{x^n-y^n}{x-y}=x^{n-1}+x^{n-2}y+\ldots.+y^{n-1}]$$

$$=\frac{1}{x-y}\{(x^2+x^3+\ldots..to\ \infty)-(y^2+y^3+\ldots..to\ \infty)\}$$

$$=\frac{1}{x-y}\left\{\frac{x^2}{1-x}-\frac{y^2}{1-y}\right\}$$

$$=\frac{1}{x-y}\left\{\frac{x^2-x^2y-y^2+xy^2}{(1-x)(1-y)}-\right\}$$

$$=\frac{x+y-xy}{(1-x)(1-y)}$$

The series can be written as:

$$3\left(\frac{1}{5}+\frac{1}{5^3}+\frac{1}{5^5}+\cdots n \text{ terms}\right)+4\left(\frac{1}{5^2}+\frac{1}{5^4}+\frac{1}{5^6}+\cdots n \text{ terms}\right)$$

For the first part $a=\frac{1}{5}$ and the common ratio $r=\frac{1}{5^2}=\frac{1}{25}$

Thus the sum is:

$$3\left(\frac{1}{5}+\frac{1}{5^3}+\frac{1}{5^5}+\cdots n \text{ terms}\right)=3\cdot\frac{\frac{1}{5}\left(1-\left(\frac{1}{25}\right)^n\right)}{1-\frac{1}{25}}$$

$$=\frac{5}{8}\left(1-\frac{1}{5^{2n}}\right)$$

For the second part $a=\frac{1}{25}$ and common ratio $r=\frac{1}{25}$ then

$$4\left(\frac{1}{5^2}+\frac{1}{5^4}+\frac{1}{5^6}+\cdots n \text{ terms}\right)=4\cdot\frac{\frac{1}{25}\left(1-\left(\frac{1}{25}\right)^n\right)}{1-\frac{1}{25}}$$

$$=\frac{1}{6}\left(1-\frac{1}{5^{2n}}\right)$$

Thus the sum is:

$$\frac{3}{5}+\frac{4}{5^2}+\frac{3}{5^3}+\cdots 2n \text{ terms}=\frac{5}{8}\left(1-\frac{1}{5^{2n}}\right)+\frac{1}{6}\left(1-\frac{1}{5^{2n}}\right)$$

$$\frac{a}{1+i}+\frac{a}{(1+i)^2}+\frac{a}{(1+i)^3}+\ldots+\frac{a}{(1+i)^n}$$

$$a=\frac{a}{1+i},\quad r=\frac{\frac{a}{(1+i)^2}}{\frac{a}{1+i}}=\frac{1}{1+i}$$

$$S_n=a\frac{\left(1-r^n\right)}{1-r}$$

$$=\frac{a}{1+i}\frac{\left(1-\left(\frac{1}{1+i}\right)^n\right)}{1-\frac{1}{1+i}}$$

$$=\frac{a}{1+i}\times\frac{1+i}{(-i)}\left(1-(1+i)^n\right)$$

$$=-ai\left(1-(1+i)^{-n}\right)$$

Re writing the sequence and sum we get,

Sum$=1-a+a^2-a^3+a^4-a^5+$..........

Here, r $=-a$ and first term =1

Sum $=\frac{[1-(-a)^n]}{1+a}$

Here the first term of the G.P is $a = x^3$ and the common ratio is $r = \frac{x^5}{x^3} = x^2$

Thus the sum of the G.P is:

$$x^3 + x^5 + x^7 + \cdots \text{ to } n \text{ terms} = \frac{x^3\left(\left(x^2\right)^n - 1\right)}{x^2 - 1} = \frac{x^3\left(x^{2n} - 1\right)}{x^2 - 1}$$

Here the first term of the G.P is $a = \sqrt{7}$ and the common ratio is $r = \frac{\sqrt{21}}{\sqrt{7}} = \sqrt{3}$

Thus the sum of the G.P is:

$$\sqrt{7} + \sqrt{21} + 3\sqrt{7} + \cdots \text{ to } n \text{ terms} = \frac{\sqrt{7}\left(\left(\sqrt{3}\right)^n - 1\right)}{\sqrt{3} - 1} = \frac{\sqrt{7}\left(3^{\frac{n}{2}} - 1\right)}{\sqrt{3} - 1}$$

Geometric Progressions Ex 20.3 Q 3

$$\sum_{n=1}^{11}\left(2+3^n\right)$$
$$=\left(2+3^1\right)+\left(2+3^2\right)+\left(2+3^3\right)+\ldots+\left(2+3^{11}\right)$$
$$=2\times 11+3^1+3^2+3^3+\ldots+3^{11}$$
$$=22+\frac{3\left(3^{11}-1\right)}{(3-1)}$$
$$=22+\frac{3\left(3^{11}-1\right)}{2}$$
$$=\frac{44+3(177147-1)}{2}$$
$$=\frac{44+3(177146)}{2}$$
$$=265741$$

So,

$$\sum_{n=1}^{11}\left(2+3^n\right)=265741$$

$$\sum_{k=1}^{n}\left(2^k+3^{k-1}\right)$$

$$=\left(2+3^0\right)+\left(2^2+3\right)+\left(2^3+3^2\right)+\ldots+\left(2^n+3^{n-1}\right)$$
$$=\left(2+2^2+2^3+\ldots+2^n\right)+\left(3^0+3^1+3^2+\ldots+3^{n-1}\right)$$
$$=S_n+S_m$$

$$S_n \Rightarrow a=2,\ n=n,\ r=\frac{2^2}{2}=2$$

$$s_n=\frac{a\left(r^n-1\right)}{r-1}=\frac{2\left(2^n-1\right)}{2-1}=2\left(2^n-1\right)$$

Also, $S_m=S_{n-1}$

$$a=1,\ r=3,\ n=n-1$$

$$S_{n-1}=\frac{1\left(3^{n-1}-1\right)}{3-1}=\frac{1}{2}\left(3^n-1\right)$$

$$\therefore \quad \sum_{k=1}^{n}\left(2^k+3^{k-1}\right)=2\left(2^n-1\right)+\frac{1}{2}\left(3^n-1\right)$$
$$=\frac{1}{2}\left[2^{n+2}+3^n-4-1\right]$$
$$=\frac{1}{2}\left[2^{n+2}+3^n-5\right]$$

$$\sum_{n=2}^{10} 4^n$$

$$= 4^2 + 4^3 + 4^4 + \ldots + 4^{10}$$

$$a = 4^2,\ r = \frac{4^3}{4} = 4,\ n = 9$$

$$S_{10} = \frac{a\left(r^9 - 1\right)}{1 - r}$$

$$= \frac{4^2\left(4^9 - 1\right)}{4 - 1}$$

$$= \frac{1}{3}\left[4^{11} - 16\right]$$

$$= \frac{16}{3}\left[4^9 - 1\right]$$

Geometric Progressions Ex 20.3 Q 4

$5+55+555+\ldots n$ terms

Taking 5 common from each term.

$$5[1+11+111+\ldots n \text{ terms}]$$

Dividing and multiplying by 9

$$=\frac{5}{9}[9+99+999+\ldots n \text{ terms}]$$

$$=\frac{5}{9}\left[(10-1)+\left(10^2-1\right)+\left(10^3-1\right)+\ldots n \text{ terms}\right]$$

$$=\frac{5}{9}\left[\left(10+10^2+10^3+\ldots n \text{ terms}\right)-n\right] \text{ this is G.P.}$$

So, $S_n=\dfrac{a\left(r^n-1\right)}{r-1}$

$a=10,\ r=10,\ n=n$

$$=\frac{5}{9}\left[\frac{10\left(10^n-1\right)}{10-1}-n\right]$$

$$=\frac{5}{9\times 9}\left(10^{n+1}-10-9n\right)$$

$$=\frac{5}{81}\left(10^{n+1}-9n-10\right)$$

Now we have

$$7+77+777+\cdots \text{ to } n \text{ terms}=7[1+11+111+\cdots \text{ to } n \text{ terms}]$$

$$=\frac{7}{9}[9+99+999+\cdots \text{ to } n \text{ terms}]$$

$$=\frac{7}{9}\left[(10-1)+\left(10^2-1\right)+\left(10^3-1\right)+\cdots \text{ to } n \text{ terms}\right]$$

$$=\frac{7}{9}\left[10+10^2+10^3+\cdots \text{ to } n \text{ terms}\right]-\frac{7}{9}(1+1+1+\cdots\cdots \text{ to } n \text{ terms})$$

$$=\frac{7}{9}\cdot\frac{10\left(10^n-1\right)}{10-1}-\frac{7n}{9}$$

$$=\frac{7}{81}\left(10^{n+1}-9n-10\right)$$

$9+99+999+\ldots n$ term

This can be written as

$$=(10-1)+(100-1)+(1000-1)+\ldots n \text{ term}$$

$$=\left(10+10^2+10^3+\ldots n \text{ term}\right)-n$$

$$\Rightarrow \quad S_n=\frac{a\left(r^n-1\right)}{r-1},\ a=10,\ r=10,\ n=n$$

$$=\frac{10\left(10^n-1\right)}{10-1}-n$$

$$=\frac{10}{9}\left(10^n-1\right)-n$$

$$=\frac{1}{9}\left[10^{n+1}-10-9n\right]$$

$$=\frac{1}{9}\left[10^{n+1}-9n-10\right]$$

$0.5+0.55+0.555+\&..$ to n

$$=5\times0.1+5\times0.11+5\times0.111+\ldots\ldots$$

$$=\frac{5}{9}\left\{\frac{9}{10}+\frac{99}{100}+\frac{999}{1000}+\ldots\ldots\ldots+-\right\}$$

$$=\frac{5}{9}\{(1-\frac{1}{10})+(1-\frac{1}{100})+\ldots\ldots\ldots+\ \}$$

$$=\frac{5}{9}\left\{n-\left(\frac{1}{10}+\frac{1}{10^2}+\ldots\ldots+\frac{1}{10^n}\right)\right\}$$

$$=\frac{5}{9}\left[n-\frac{1}{10}\frac{\left\{1-(\frac{1}{10})^n\right\}}{\left(1-\frac{1}{10}\right)}\right]$$

$$=\frac{5}{9}\left[n-\frac{1}{9}(1-\frac{1}{10^n})\right]$$

$0.6+0.66+0.666+\&..$ to n

$$=6\times0.1+6\times0.11+6\times0.111+\ldots\ldots$$

$$=\frac{6}{9}\left\{\frac{9}{10}+\frac{99}{100}+\frac{999}{1000}+\ldots\ldots\ldots+-\right\}$$

$$=\frac{6}{9}\{(1-\frac{1}{10})+(1-\frac{1}{100})+\ldots\ldots\ldots+\ \}$$

$$=\frac{6}{9}\left\{n-\left(\frac{1}{10}+\frac{1}{10^2}+\ldots\ldots+\frac{1}{10^n}\right)\right\}$$

$$=\frac{6}{9}\left[n-\frac{1}{10}\frac{\left\{1-(\frac{1}{10})^n\right\}}{\left(1-\frac{1}{10}\right)}\right]$$

$$=\frac{6}{9}\left[n-\frac{1}{9}(1-\frac{1}{10^n})\right]$$

Geometric Progressions Ex 20.3 Q 5

Here,

$$3,\ \frac{3}{2},\ \frac{3}{4},\ \ldots \text{ is a G.P.}$$

and $$S_n = \frac{3069}{512},\ a = 3,\ r = \frac{1}{2}$$

$$S_n = \frac{a\left(1-r^n\right)}{1-r}$$

$$\frac{3069}{512} = \frac{3\left(1-\left(\frac{1}{2}\right)^n\right)}{1-\frac{1}{2}}$$

$$\frac{3069}{512} = \frac{3\left(2^n - 1\right)}{2^n \times \frac{1}{2}}$$

$$\frac{1023}{512} = \frac{2\left(2^n - 1\right)}{2^n}$$

$$10232^n = 1024.2^n - 1024$$

$$1024 = 2^n$$

$\Rightarrow$ $$2^{10} = 2^n$$

$\Rightarrow$ $$n = 10$$

Geometric Progressions Ex 20.3 Q 6

$2 + 6 + 18 + \ldots$

$$S_n = 728$$

Now,

$$S_n = \frac{a\left(r^n - 1\right)}{r-1}$$

$$a = 2,\ r = \frac{6}{2} = 3$$

$$728 = \frac{2\left(3^n - 1\right)}{3-1}$$

$$728 = \frac{2\left(3^n - 1\right)}{2} = \left(3^n - 1\right)$$

$$728 + 1 = 3^n$$

$$729 = 3^n$$

$$(3)^6 = 3^n$$

$\Rightarrow$ $$n = 6$$

Geometric Progressions Ex 20.3 Q 7

$\sqrt{3}, 3, 3\sqrt{3}, \ldots$

$$S_n = \frac{a\left(r^n - 1\right)}{r - 1}$$

$$a = \sqrt{3},\ r = \frac{3}{\sqrt{3}} = \sqrt{3},\ S_n = 39 + 13\sqrt{3}$$

Putting into formula

$$39 + 13\sqrt{3} = \frac{\sqrt{3}\left(\left(\sqrt{3}\right)^n - 1\right)}{\sqrt{3} - 1}$$

$$39 + 13\sqrt{3} = \frac{\left(\sqrt{3}\right)^{n+1} - \sqrt{3}}{\sqrt{3} - 1}$$

$$\left(39 + 13\sqrt{3}\right)\left(\sqrt{3} - 1\right) = \left(\sqrt{3}\right)^{n+1} - \sqrt{3}$$

$$39\sqrt{3} - 39 + 39 - 13\sqrt{3} = \left(\sqrt{3}\right)^{n+1} - \sqrt{3}$$

$$26\sqrt{3} + \sqrt{3} = \left(\sqrt{3}\right)^{n+1}$$

$$\left(27\sqrt{3}\right)^1 = \left(\sqrt{3}\right)^{n+1}$$

$$\left(\sqrt{3}\right)^6\left(\sqrt{3}\right)^1 = \left(\sqrt{3}\right)^{n+1}$$

$$7 = n + 1$$

$$\Rightarrow \quad n = 6$$

Geometric Progressions Ex 20.3 Q 8

$3, 6, 12, \ldots n\ 381$

$$a = 3,\ r = \frac{6}{3} = 2,\ n = ?\ S_n = 381$$

We know that

$$S_n = \frac{a\left(r^n - 1\right)}{r - 1}$$

$$381 = \frac{3(2)^n - 1}{2 - 1}$$

$$\frac{381}{3} = 2^n - 1$$

$$127 = 2^n - 1$$

$$128 = 2^n$$

$$2^7 = 2^n$$

$$n = 7$$

Geometric Progressions Ex 20.3 Q 9

$r = 3$, last term is 486

Sum of terms $= S_n = 728$, $a = ?$

We know that

$$S_n = \frac{a\left(r^n - 1\right)}{r - 1}$$

$$728 = \frac{a\left(3^n - 1\right)}{3 - 1}$$

Also, $t_n = ar^{n-1}$

$t_n = 486$

$\therefore$ $486 = a(3)^{n-1}$

$a\left(3^{n-1}\right) = 3^5 \times 2$

$3^{n-1} = 3^5$

$n = 6$

and $a = 2$

Geometric Progressions Ex 20.3 Q 10

Let Sum of first three terms $= a + ar + ar^2$

$$\text{The ratio} = \frac{a + ar + ar^2}{a + ar + ar^2 + ar^3 + ar^4 + ar^5}$$

$$= \frac{1 + r + r^2}{1 + r + r^2 + r^3 + r^4 + r^5}$$

$$= \frac{1 + r + r^2}{1 + r + r^2 + r^3(1 + r + r^2)} \text{.......................(1)}$$

Let $A = 1 + r + r^2$ ....(2)

$$Ratio = \frac{A}{A + r^3 A} = \frac{125}{152}$$

$$\frac{1}{1 + r^3} = \frac{125}{152}$$

$$152 = 125 + 125\,r^3$$

$$r^3 = \frac{27}{125}$$

$$r = \frac{3}{5}$$

Geometric Progressions Ex 20.3 Q 11

$t_4 = \frac{1}{27}, \; t_7 = \frac{1}{729}, \; t_n = ar^{n-1}$

Where $t_n = n^{th}$ term, r = common difference, n = number of terms.

$$t_4 = ar^3 = \frac{1}{27} \quad \text{---(i)}$$

$$t_7 = ar^6 = \frac{1}{729} \quad \text{---(ii)}$$

Dividing(ii) by (i), we get

$$\frac{t_7}{t_4} = \frac{ar^6}{ar^3} = r^3 = \frac{27}{729} = \frac{1}{27}, \; r = \frac{1}{3}$$

Sum of n term $= S_n = \frac{a\left(1-r^n\right)}{1-r}$ ---(i)

When, $r = 3$, $t_4 = ar^3 = \frac{1}{27}$

$$a\left(\frac{1}{3}\right)^3 = \frac{1}{27}$$

$$a = 1$$

Substituting $a = 1$, $r = \frac{1}{3}$ in(i)

$$S_n = \frac{1\left(1-\left(\frac{1}{3}\right)^n\right)}{1-\frac{1}{3}}$$

$$= \frac{1-\left(\frac{1}{3}\right)^n}{\frac{2}{3}}$$

$$= \frac{3}{2}\left(1-\left(\frac{1}{3}\right)^n\right)$$

Geometric Progressions Ex 20.3 Q 12

$$\sum_{n=1}^{10}\left\{\left(\frac{1}{2}\right)^{n-1}+\left(\frac{1}{5}\right)^{n+1}\right\}$$

$$= \sum_{n=1}^{10}\left(\frac{1}{2}\right)^{n-1} + \sum_{n=1}^{10}\left(\frac{1}{5}\right)^{n+1}$$

$$= 1+\frac{1}{2}+\frac{1}{2^2}+\ldots\ldots+\frac{1}{5^2}+\frac{1}{5^3}+\frac{1}{5^4}+\ldots\ldots$$

$$= \frac{(1-\frac{1}{2^{10}})}{1-\frac{1}{2}}+\frac{\frac{1}{5}(1-\frac{1}{5^{10}})}{1-\frac{1}{5}}$$

$$= \frac{2^{10}-1}{2^9}+\frac{5^{10}-1}{5^{11}}$$

Geometric Progressions Ex 20.3 Q 13

Fifth term of series is

$ar^{5-1} = 81 ....(1)$

Second term of series is

$ar = 24 \quad ....(2)$

Dividing (2) by (1) we get,

$$\frac{ar}{ar^4} = \frac{24}{81} = \frac{8}{27}$$

$$r^3 = \frac{27}{8}$$

$$r = \frac{3}{2}$$

Substituting r in (2), we get,

$$a = \frac{24 \times 2}{3} = 16$$

$$Sum = \frac{16\left[(\frac{3}{2})^8 - 1\right]}{\frac{3}{2} - 1}$$

$$= \frac{16\left[3^8 - 2^8\right]}{2^7}$$

$$= \frac{6305}{8}$$

Geometric Progressions Ex 20.3 Q14

S_1 = sum of n terms,
S_1 = sum of $2n$ terms,
S_1 = sum of $3n$ terms.

Then, $S_1^2 + S_2^2$

$$= (S_n)^2 + (S_{2n})^2$$
$$= \left(\frac{a(1-r^n)}{1-r}\right)^2 + \left(\frac{a(1-r^{2n})}{1-r}\right)^2$$
$$= \frac{a^2}{(1-r)^2}\left[\left(1-(r)^n\right)^2 + \left(1-r^{2n}\right)^2\right]$$
$$= \frac{a^2}{(1-r)^2}\left[1+r^{2n}-2r^n+1+r^{4n}-2r^{2n}\right]$$
$$= \frac{a^2}{(1-r)^2}\left[2-r^{2n}-2r^n+r^{4n}\right] \quad \text{---(i)}$$

Also, $S_1(S_2+S_3)$

$$= \frac{a(1-r^n)}{1-r}\left(\frac{a(1-r^{2n})}{1-r} + \frac{a(1-r^{3n})}{1-r}\right)$$
$$= \frac{a^2}{(1-r)^2}\left[(1-r)^n\left(1-r^{2n}\right) + \left(1-r^n\right)\left(1-r^{3n}\right)\right]$$
$$= \frac{a^2}{(1-r)^2}\left[1-r^{2n}-r^n+r^{3n}-r^{3n}-r^n+1+r^{4n}\right]$$
$$= \frac{a^2}{(1-r)^2}\left[2-r^{2n}-2r^n+r^{4n}\right] \quad \text{---(ii)}$$

(i) = (ii) Hence, $S_1^2 + S_2^2 = S_1(S_2+S_3)$

Geometric Progressions Ex 20.3 Q15

$S_1, S_2, \ldots, S_n$ are the sums of n terms of G.P. $a = 1$, $r = 1, 2, 3, \ldots, n$

Then, $S_1 + S_2 + 2S_3 + 3S_4 + \ldots + (n-1)S_n$

$$\frac{1(1^n-1)}{1-1} + \frac{1(2^n-1)}{2-1} + \frac{2(3^n-1)}{3-1} + \ldots + (n-1)1\left(\frac{1^n-1}{1-1}\right)$$
$$= 2^n - 1 + 23^n - 1 + 3.4^n - 1 + \ldots$$
$$= 2^n + 3^n + 4^n + \ldots + n^n$$

Geometric Progressions Ex 20.3 Q16.

Let the G.P. be $2n, 2, 2n+4, \dots$

Then, $S_n = \dfrac{a\left(r^n - 1\right)}{r-1}$, $a = 2n$, $r = 2$

$\therefore \quad S_n = \dfrac{2n\left(2^n - 1\right)}{2-1} = 2n^{n+1} - 2n$

Then the G.P. of odd term

$a_1 + a_3 + a_5 + \dots a_{2n-1}$

According to the question

Sum of all terms = 5 (sum of terms occupying the odd places)

$a_1 + a_2 + a_3 + \dots + a_{2n} = 5\left(a_1 + a_3 + a_5 + \dots + a_{2n-1}\right)$

$a + ar + ar^2 + \dots + ar^{2n-1} = 5\left(a + ar^2 + ar^4 + \dots + ar^{2n-2}\right)$

$$\frac{a\left(1-r^{2n}\right)}{1-r} = 5\left(\frac{a\left(1-\left(r\right)^2\right)^n}{1-r^2}\right)$$

$\dfrac{a}{1-r}$ is cancelled on both side

$1 - r^{2n} = \dfrac{5\left(1-r^{2n}\right)}{1+r}$

$1 + r - r^{2n} - r^{2n+1} = 5 - 5r^{2n}$

$r^{2n+1} - 4r^{2n} - r + 4 = 0$

$r^{2n}\left(r-4\right) - 1\left(r-4\right) = 0$

$r^{2n} = 1,\ r = 4$

$\Rightarrow \quad r = 4$

Geometric Progressions Ex 20.3 Q17

Given $\sum_{n=1}^{100} a_{2n} = \alpha$

$\Rightarrow \quad a_2 + a_4 + a_6 + \dots + a_{200} = \alpha$ ---(i)

also, $\sum_{n=1}^{100} a_{2n-1} = \beta$

$\Rightarrow \quad a_1 + a_3 + a_5 + \dots + a_{199} = \beta$ ---(ii)

Sum of G.P,

$S_n = \dfrac{a\left(1-r^n\right)}{1-n}$

$= a = a_2, r = r^2, n = 100$

$ar + ar^3 + ar^5 + \dots + ar^{199} = \alpha$

$ar\dfrac{\left(1-\left(r^2\right)^{100}\right)}{1-r^2} = \alpha$ ---(iii)

$a + ar^2 + ar^4 + \dots + ar^{198} = \beta$

$\dfrac{a\left(1-\left(r^2\right)^{100}\right)}{1-r^2} = \beta$ ---(iv)

$r\left(\beta\right) = \alpha$

$r = \dfrac{\alpha}{\beta}$ [From (iv) and (v)]

Geometric Progressions Ex 20.3 18

Let the seried be $a_1 + a_2 + a_3 + \ldots + a_{2n}$

It is given that $a_1 = 1, a_2 = a, a_3 = ac, a_4 = a^2c, a_5 = a^2c^2, \ldots$

$\therefore$ Sum of $2n$ term

$$a_1 + a_2 + a_3 + \ldots + a_{2n}$$

$$= 1 + a + ac + a^2c + a^2c^2 + \ldots + 2n \text{ term}$$

$$= (1+a) + ac(1+a) + a^2c^2(1+a) + \ldots + n \text{ term}$$

$$= (1+a)\frac{\left(1-(ac)^n\right)}{1-ac}$$

$$= (a+1)\frac{\left((ac)^n - 1\right)}{ac-1}.$$

Geometric Progressions Ex 20.3 Q19.

Sum of first n term of G.P.

$$= a + a_2 + a_3 + \ldots + a_n$$

$$= a + ar + ar^2 + \ldots + ar^{n-1} \qquad \left[\because t_n = ar^{n-1}\right] \text{ ---(i)}$$

Also sum of term from

$(n+1)^{th}$ to $(2n)^{th}$ term is

$$= a_{n+1} + a_{n+2} + \ldots + a_{2n}$$

$$= ar^n + ar^{n-1} + \ldots + ar^{2n-1} \qquad \text{---(ii)}$$

Ratio of (i) and (ii) is

$$= \frac{a + ar + ar^2 + \ldots ar^{n-1}}{ar^n + ar^{n-1} + \ldots + ar^{2n-1}} \qquad \left[\because S_n = \frac{a\left(1-r^n\right)}{1-r}\right]$$

$$= \frac{\frac{a\left(1-r^n\right)}{1-r}}{\frac{ar^n\left(1-r^n\right)}{1-r}}$$

$$= \frac{1}{r^n}$$

Geometric Progressions Ex 20.3 Q20

Given,

$a,\ b$ are roots of the equation $x^2 - 3x + p = 0$

$\Rightarrow \quad a + b = 3,\ ab = p$

and $\quad c,\ d$ are roots of the equation $x^2 - 12x + q = 0$

$\Rightarrow \quad c + d = 12,\ cd = q$

Let $b = ar,\ c = ar^2$ and $d = ar^3$, then $a + b = 3$ and $c + d = 12$

$a(1+r) = 3$ and $ar^2(1+r) = 12$

$$\Rightarrow \quad \frac{ar^2(1+r)}{a(1+r)} = \frac{12}{3}$$

$\Rightarrow \quad r = 2$

and $a(r+1) = 3$

$\Rightarrow \quad a = 1$

$p = ab$

$= a \times ar$

$p = 2$

$q = cd$

$= ar^2 \times ar^3$

$= 2^5$

$a = 32$

$$\frac{q+p}{q-p} = \frac{32+2}{32-2}$$

$$= \frac{34}{30}$$

$(q+p) : (q-p) = 17 : 15$

Geometric Progressions Ex 20.3 Q21.

$$\text{Sum} = \frac{3069}{512} = \frac{3(1-\frac{1}{2^n})}{\frac{1}{2}}$$

$$1-\frac{1}{2^n} = \frac{3069}{512\times6} = \frac{1023}{512\times2}$$

$$1-\frac{1023}{1024} = \frac{1}{2^n}$$

$$\frac{1}{2^n} = \frac{1}{1024}$$

$n = 10$

Geometric Progressions Ex 20.3 Q22.

To find number of ancestors, we will find the sum of $2, 2^2, 2^3, \ldots$

$$\text{Number of ancestors} = \frac{2(2^{10}-1)}{2-1}$$

$= 2(1024-1)$

$= 2\times1023$

$= 2046$