RD Sharma Solutions Class 11 Maths Chapter 19 Ex 19.7

Arithematic Progressions Ex 19.7 Q1

Let the amount saved by the man in first year be x.

Then,

ATQ

$x+(x+100)+(x+200)+\ldots+(x+900)=16500$

As his saving increased by Rs 100 every year.

$\therefore \quad 10x+100+200+\ldots+900=16500 \qquad$ ---(i)

Here,

$100+200+300+\ldots+900$ form a seried of

$a=100,\ d=100$ and $n=9$

So,

$$S_n=\frac{n}{2}[a+l]$$

$$S_9=\frac{9}{2}[100+900]=4500 \qquad \text{---(ii)}$$

From (i) and (ii)

$10x+(4500)=16500$

$10x=12000$

or $\quad x=1200$

The man saved Rs 1200 in the first year.

Arithematic Progressions Ex 19.7 Q2

Let the man save Rs 200 in n numbers of years.

Then,

ATQ

$32+36+40+\ldots=200$

It rorms a series of n terms, with $a=32$ and $d=4$

$$\therefore \quad S_n=\frac{n}{2}[2a+(n-1)d]$$

$$\Rightarrow \quad 200=\frac{n}{2}[2(32)+(n-1)4]$$

$\Rightarrow \quad 400=60n+4n^2$

$\Rightarrow \quad n^2+15n-100=0$

$\Rightarrow \quad n=5$ or -20

But, $n \neq -20$ [It can't be negative]

$\therefore$ $n = 5$

The man will save Rs 200 in 5 years.

Arithematic Progressions Ex 19.7 Q3

Let the 40 annual instalments form an alithmetic series of common diference d and first instalment a

Then, series so firmed is

$$a + (a + d) + (a + 2d) + \ldots = 3600$$

or $s_n = \frac{n}{2}[2a + (n-1)d]$

or $3600 = 20[2a + 39d]$

$2a + 39d = 180$ ---(i)

and sum of first 30 terms is $\frac{2}{3}$ of 3600

$= 2400$

$\Rightarrow$ $2400 = \frac{30}{2}[2a + (29)d]$

or $2a + 29d = 160$ ---(ii)

From (i) and (ii)

$a = 51$

The first installment paid by this man is Rs 51.

Arithematic Progressions Ex 19.7 Q4

Let the number of Radio manufactured increase by x each year and number of radio manufacture in first year be a. So, A.P formed ATQ is

$$a, a + x, a + 2x, \ldots$$

Here,

$a_3 = a + 2x = 600$ ---(i)

$a_7 = a + 6x = 700$ ---(ii)

From (i) and (ii)

$a = 550,\ x = 25$

(i) 550 Radio's were manufactured in the first year.

(ii) The total produce in 7 years is sum of produce in the first 7 years.

$$S_7 = \frac{7}{2}[550 + 700] \qquad \left[\because S_n = \frac{n}{2}[a + l]\right]$$

$$= 4375$$

4375 Radio's were manufactured in first 7 years.

(iii) The product in 10th year

$$a_{10} = a + 9d$$

$$= 550 + 9(25) = 775$$

775 Radio's were manufactured in the 10th year.

Arithematic Progressions Ex 19.7 Q5

There are 25 trees at equal distance of 5 m in a line with a well(w), and the distance of the well from the nearesst tree = 10 m.

Thus,

The total distance travelled by gardener to tree 1 and back is 2×10 m = 20 m

Similarly for all the 25 trees.

The distance covered by gardener is

$$= 2\left[10 + (10 + 5) + (10 + 2 \times 5) + (10 + 3 \times 5) + \ldots + (10 + 23 \times 5)\right] \qquad \text{---(i)}$$

This forms a series of 1st term $a = 10$, common difference $d = 5$ and $n = 25$

$\therefore \qquad 10 + (10 + 5) + (10 + 2 \times 5) + \ldots + (10 + 24 \times 5)$

$$\Rightarrow \qquad S_{25} = \frac{25}{2}\left[2 \times 10 + (24)5\right] = 25\left[10 + 60\right] = 1750 \text{ m} \qquad \text{---(ii)}$$

From (i) and (ii)

Total distance = 2×1750 m = 3500 m.

Arithematic Progressions Ex 19.7 Q6

The man counts at the rate of Rs 180 per minute for half an hour. After this he counts at the rate of Rs 3 less every minute than preceding minute.

Then, the amount counted in first 30 minute

$$= \text{Rs } 180 \times 30 = \text{Rs } 5400 \qquad \text{---(i)}$$

The amount left to be counted after 30 minute

$= \text{Rs } 10710 - 5400 = \text{Rs } 5310$ ---(ii)

ATQ

A.P formed is $(180 - 3) + (180 - 2 \times 3) + \ldots = 5310$

Let time taken to count 5310 be t

Then,

$$S_t = \frac{t}{2}\left[(180 - 3) + (t - 1)(-3)\right]$$

$$5310 = \frac{t}{2}\left[200 - 3t\right]$$

or $t = 59$ minute

Thus, the total time taken by the man to count Rs 10710 is $(59 + 30) = 89$ minutes.

Arithematic Progressions Ex 19.7 Q7

The piece of equipment deprecites 15% in first year i.e., $\frac{15}{100} \times 600,000 = \text{Rs } 90,000$

$\therefore$ Value after 1st year $= 600,000 - 90,000$

$= \text{Rs } 510,000$ ---(i)

The equipment deprecites at the rate 135% in 2nd year i.e., $\frac{135}{1000} \times 600,000 = 81000$

$\therefore$ Value after 2nd year $= 81000$

The value after 3rd year $= \frac{12}{100} \times 600000 = 72000$

The total depreciation in 10 years

$$\Rightarrow \quad S_{10} = \frac{10}{2}\left[2 \times 81000 + (9)(-9000)\right]$$

$$= 5\left[81000\right] \qquad \left[\text{Using } S_n = \frac{n}{2}\left[2a + (n - 1)d\right]\right]$$

$$= 405000$$

$\therefore$ The cost of machine after 10 years = Rs 600000 − 405000

$$= 105000.$$

Arithematic Progressions Ex 19.7 Q8

Total cost of tractor

$$= 6000 + \left[(500 + 12\% \text{ of } 6000 \text{ for } 1 \text{ year}) + (500 + 12\% \text{ of } 5500 \ 1 \text{ year}) + \ldots .. + 12 \text{ times}\right]$$

$$= 6000 + 6000 + \frac{12}{100}(6000 + 5500 + \ldots \ldots + 12 \text{ times})$$

$$= 12000 + \frac{12}{100}\left[\frac{12}{2}(6000 + 5000)\right]$$

$$= 12000 + \frac{12}{100} \times \frac{12}{2} \times 6500$$

$$= 12000 + (72 \times 65)$$

$$= 12000 + 4680$$

$$= 16680$$

Total cost of tractor = Rs. 16680

Arithematic Progressions Ex 19.7 Q9

Total cost of Scooter

$$= \text{Rs}4000 + \begin{bmatrix} \{\text{Rs } 1000 + \text{S.I. on Rs Rs } 18000 \text{ for } 1 \text{ year}\} \\ +\{\text{Rs } 1000 + \text{S.I. on Rs Rs } 17000 \text{ for } 1 \text{ year}\} \\ +\ldots .. + 18 \text{ times} \end{bmatrix}$$

$$= (4000 + 18000) + \text{S.I. for } 1 \text{ year on } (18000 + 17000 + \ldots ..\text{to } 18 \text{ times})$$

$$= 22000 + \text{S.I. for } 1 \text{ year on } \left\{\frac{18}{2}(18000 + 1000)\right\}$$

$$= 22000 + 9(19000) \times \frac{10}{100}$$

$$= 22000 + 17100$$

$$= \text{Rs } 39100$$

Total cost of Scooter = Rs. 39100

Arithematic Progressions Ex 19.7 Q10

First year the person income is: 300,000

Second year his income will be: $300,000+10,000=310,000$

⋮

⋮

This way he receives the amount after 20 years will be:

$$300,000+310,000+\cdots+490,000$$

This is an AP with first term $a=300000$ and common difference $d=10,000$.

Therefore

$$S=\frac{20}{2}\left[2\cdot 300000+(20-1)10000\right]$$

$$=10\left[600000+190000\right]$$

$$=7900000$$

Arithematic Progressions Ex 19.7 Q11

In 1^{st} installment the man paid 100 rupees.

In 2^{nd} installment the man paid $(100+5)=105$ rupees.

⋮

⋮

Likewise he pays up to the 30^{th} installment as follows:

$$100+105+\cdots+(100+5\times 29)$$

This is an AP with $a=100$ and common difference $d=5$.

Therefore at the 30^{th} installment the amount he will pay

$$T_{30}=100+(30-1)(5)$$

$$=100+145$$

$$=245$$

Arithematic Progressions Ex 19.7 Q12

Suppose carpenter took n days to finish his job.

First day carpenter made five frames

$a_1 = 5$

Each day after first day he made two more frames
$d=2$

$\therefore$ On n^{th} day frames made by carpenter are,
$a_n = a_1 + (n-1)d$
$\Rightarrow a_n = 5 + (n-1)2$

Sum of all the frames till n^{th} day is
$S = \frac{n}{2}[a_1 + a_n]$
$192 = \frac{n}{2}[5 + 5 + (n-1)2]$
$192 = 5n + n^2 - n$
$n^2 + 4n - 192 = 0$
$(n+16)(n-12) = 0$
$n = -16$ or $n = 12$

But number of days cannot be negative hence $n = 12$.

The carpenter took 12 days to finish his job.

Arithematic Progressions Ex 19.7 Q13

We know that sum of interior angles of a polygon with n sides is given by,
$a_n = 180°(n - 2)$

Sum of interior angles of a polygon with 3 sides is given by,
$a_3 = 180°(3 - 2) = 180°$...........(i)

Sum of interior angles of a polygon with 7 sides is given by,
$a_4 = 180°(4 - 2) = 360°$...........(ii)

Sum of interior angles of a polygon with 5 sides is given by,
$a_5 = 180°(5 - 2) = 540°$...........(iii)

From eq^n (i), eq^n (ii) and eq^n (iii) we get,
$a_4 = 360° = 180° + 180° = a_3 + 180° = a_3 + d$
$a_5 = 540° = 180° + 360° = a_3 + 2d$

Hence the sums of the interior angles of polygons with 3, 4, 5, 6,... sides form an arithmetic progression.

Sum of interior angles of 21 sided polygon
$= 180°(21 - 2)$
$= 3420°$