RD Sharma Solutions Class 11 Maths Chapter 19 Ex 19.6

Arithematic Progressions Ex 19.6 Q1

(i) 7 and 13

Let A be the arithematic mean of 7 and 13.

Then,

$$7, A, 13 \text{ ar ein A.P}$$

$$\Rightarrow \quad A - 7 = 13 - A$$

$$\Rightarrow \quad A = \frac{13 + 7}{2} = 10$$

$\therefore$ A.M is 10.

(ii) 12 and -8

Let A be the arithematic mean of 12 and -8

Then,

$$12, A, -8 \text{ are in A.P}$$

$$\Rightarrow \quad A - 12 = -8 - A$$

$$\Rightarrow \quad A = \frac{12 + (-8)}{2} = 2$$

$\therefore$ A.M is 2.

(iii) $(x - y)$ and $(x + y)$

Let A be the arithematic mean of $(x - y)$ and $(x + y)$

Then,

$$(x - y), A, (x + y) \text{ are in A.P}$$

$$\Rightarrow \quad A - (x - y) = (x + y) - A$$

$$\Rightarrow \quad A = \frac{(x - y) + (x + y)}{2} = \frac{2x}{2} = x$$

$\therefore$ A.M is x.

Arithematic Progressions Ex 19.6 Q2

Let A_1, A_2, A_3, A_4 be the 4 A.M.s between 4 and 19

Then,

$4, A_1, A_2, A_3, A_4, 19$ are in A.P of 6 terms

$$A_n = a + (n-1)d$$

$$a_6 = 19 = 4 + (6-1)d$$

or $\quad d = 3 \qquad$ ---(i)

Now,

$$A_1 = a + d = 4 + 3 = 7$$

$$A_2 = A_1 + d = 7 + 3 = 10$$

$$A_3 = A_2 + d = 10 + 3 = 13$$

$$A_4 = A_3 + d = 13 + 3 = 16$$

The 4 A.M.s between 4 and 19 are 7, 10, 13, 16.

Arithematic Progressions Ex 19.6 Q3

$$2, a_1, a_2, a_3, a_4, a_5, a_6, a_7, 17$$

$$17 = a + 8d$$

$$a = 2 \Rightarrow d = \frac{15}{8}$$

$$a_1 = 2 + \frac{15}{8} = \frac{31}{8}$$

$$a_2 = \frac{31}{8} + \frac{15}{8} = \frac{46}{8}$$

so we get our final series as

$$2, \frac{31}{8}, \frac{46}{8}, \frac{61}{8}, \frac{76}{8}, \frac{91}{8}, \frac{106}{8}, \frac{121}{8}, \frac{136}{8} = 17$$

Arithematic Progressions Ex 19.6 Q4

Let $A_1, A_2, A_3, A_4, A_5, A_6$ be the 6 AM's between 15 and -13

Then,

$15, A_1, A_2, A_3, A_4, A_5, A_6, -13$ are in A.P of 8 terms

Here, $-13 = a_8 = a + 7d$

$\Rightarrow$ $-13 = 15 + 7d$

or $d = -4$ ---(i)

$\therefore$ $A_1 = a + d = 15 - 4 = 11$

$A_2 = a + 2d = 15 - 2(4) = 7$

$A_3 = a + 3d = 15 - 4(3) = 3$

$A_4 = a + 4d = 15 - 4(4) = -1$

$A_5 = a + 5d = 15 - 4(5) = -5$

$A_6 = a + 6d = 15 - 4(6) = -9$

The 6 A.M.s between 15 and -13 are $11, 7, 3, -1, -5$ and -9.

Arithematic Progressions Ex 19.6 Q5

Let the n A.M's between 3 and 17 be $A_1, A_2, A_3, \ldots, A_n$

Then,

ATQ

$$\frac{A_n}{A_1} = \frac{3}{1} \quad \text{---(i)}$$

We know that

$3, A_1, A_2, A_3, \ldots, A_n, 17$ are in A.P of $n + 2$ terms

So, 17 is the $(n+2)$ th terms.

i.e. $17 = 3 + (n + 2 - 1)d$ $[\text{Using } a_n = a + (n-1)d]$

or $$d = \frac{14}{(n+1)} \quad \text{---(ii)}$$

$\therefore$ $A_n = 3 + (n + 1 - 1)d$

$$= 3 + \frac{14n}{n+1} = \frac{17n+3}{n+1} \quad \text{---(iii)}$$

$$A_1 = 3 + d = \frac{3n+17}{n+1} \quad \text{---(iv)}$$

From (i), (iii) and iv

$$\frac{A_n}{A_1} = \frac{17n+3}{3n+17} = \frac{3}{1}$$

$\therefore$ $n = 6$

There are 6 A'M between 3 and 17.

Arithematic Progressions Ex 19.6 Q6

Let there be n A.M between 7 and 71 and let the A.M's be $A_1, A_2, A_3, \ldots, A_n$.

So,

$$7, A_1, A_2, A_3, \ldots, A_n, 71 \text{ are in A.P of } (n+2) \text{ terms}$$

$$A_5 = a_6 = a + 5d = 27 \qquad \text{[Given]}$$

$$\Rightarrow \quad a + 5d = 27$$

$$\Rightarrow \quad d = 4 \qquad [\because a = 7] \qquad \text{---(i)}$$

The $(n+2)$ th term of A.P is 71

$$\therefore \quad a_{n+2} = 7 = a + (n+2-1)d$$

or $\quad n = 15$

There are 15 AM's between 7 and 71.

Arithematic Progressions Ex 19.6 Q7

Let $A_1, A_2, A_3, A_4, \ldots, A_n$ be the n AMs inserted between two number a and b.

Then,

$$a, A_1, A_2, A_3, A_4, \ldots, A_n, b \text{ are in A.P}$$

So, the mean of a and b

$$\text{A.M} = \frac{a+b}{2}$$

The mean of A_1 and A_n

$$\text{A.M} = \frac{a+d+b-d}{2} = \frac{a+b}{2}$$

Similarly mean of A_2 and A_{n-1}

$$\text{A.M} = \frac{a+2d+b-2d}{2} = \frac{a+b}{2}$$

Similarly we observe the means is equidistant from begining and the end is constant $\frac{a+b}{2}$.

The AM is $\frac{a+b}{2}$.

Arithematic Progressions Ex 19.6 Q8

Here,

A_1 is the A.M of x and y,

and A_2 is the A.M of y and z.

Then,

$$A_1 = \frac{x+y}{2} \qquad ---(i) \qquad \left[\because AM = \frac{a+b}{2}\right]$$

$$A_2 = \frac{y+z}{2} \qquad ---(ii)$$

Let A.M be the arithematic mean of A_1 and A_2

Then,

$$A.M = \frac{A_1 + A_2}{4}$$

$$= \frac{x+y+y+z}{4}$$

$$= \frac{x+2y+z}{4} \qquad ---(iii)$$

Since, 4, y, z are in A.P [Given]

$$y = \frac{x+a}{2} \qquad ---(iv)$$

From (iii) and (iv)

$$A.M = \frac{\left(\frac{x+a}{2}\right)+\left(\frac{2y}{2}\right)}{2} = \frac{y+y}{2} = y$$

Hence, proved A.M between A_1 and A_2 is y.

Arithematic Progressions Ex 19.6 Q9

$8, a_1, a_2, a_3, a_4, a_5, 26$

$a = 8$

$a + 6d = 26$

$$\Rightarrow d = \frac{18}{6} = 3$$

So series is 8, 11, 14, 17, 20, 23, 26