RD Sharma Solutions Class 11 Maths Chapter 19 Ex 19.4

Arithematic Progressions Ex 19.4 Q1

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$S_{10} = \frac{10}{2} [2 \times 50 + (10-1)(-4)]$$

$$= 320$$

$$S_{12} = \frac{12}{2} [2 \times 1 + (12 - 1)2]$$
$$= 6 \times 24 = 144$$

(iii)
$$3, \frac{9}{2}, 6, \frac{15}{2}, ..., 25 \text{ terms}$$

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$S_{25} = \frac{25}{2} (2 \times 3 + 24 \times \frac{3}{2})$$

$$= 525$$

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$S_{25} = \frac{25}{2} [2 \times 41 + (11)(-5)]$$

$$= 162$$

(v)
$$a + b, a - b, a - 3b, ...$$
 to 22 terms
$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$S_{22} = \frac{22}{2} [2a + 2b + 21(-2b)]$$
$$= 22a - 440b$$

(vi)
$$(x-y)^2$$
, (x^2+y^2) , $(x+y)^2$,..., x terms

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$= \frac{n}{2} [2(x^2 + y^2 - 2xy) + (x-1)(-2xy)]$$

$$= n[(x-y)^2 + (n-1)xy]$$

$$\frac{x-y}{x+y}$$
, $\frac{3x-2y}{x+y}$, $\frac{5x-3y}{x+y}$,....to n terms

nth term in above sequence is
$$\frac{(2n-1)x-ny}{x+y}$$

Sum of n terms is given by

$$\frac{1}{x+y} \Big[x + 3x + 5x + \dots + (2n-1)x - (y + 2y + 3y \dots + ny) \Big]$$

$$= \frac{1}{x+y} \left[\frac{n}{2} (2x + (n-1)2x) - \frac{n(n+1)y}{2} \right]$$

$$= \frac{1}{2(x+y)} \left[2n^2x - 2n^2y - ny \right]$$

Arithematic Progressions Ex 19.4 Q2

(i) 2+5+8+...+182.

a, term of given A.P is 182

$$a_n = a + (n-1)d = 182$$

$$\Rightarrow 182 = 2 + (n-1)3$$

or
$$n = 61$$

Then,

$$S_n = \frac{n}{2} [a+l]$$

$$= \frac{61}{2} [2+182]$$

$$= 61 \times 92$$

$$= 5612$$

(ii) 101+99+97+...+47

 a_n term of A.P of n terms is 47.

$$47 = a + (n-1)d$$

$$47 = 101 + (n-1)(-2)$$

or
$$n = 28$$

Then,

$$S_n = \frac{n}{2} [a+l]$$

$$= \frac{28}{2} [101+47]$$

$$= 14 \times 148$$

$$= 2072$$

(iii)
$$(a-b)^2 + (a^2 + b^2) + (a+b)^2 + ... + [(a+b)^2 + 6ab]$$

Let number of terms be n

Then,

$$a_n = (a+b)^2 + 6ab$$

$$\Rightarrow (a-b)^2 + (n-1)(2ab) = (a+b)^2 + 6ab$$

$$\Rightarrow a^2 + b^2 - 2ab + 2abn - 2ab = a^2 + b^2 + 2ab + 6ab$$

$$\Rightarrow n = 6$$
Then

Then,

$$S_n = \frac{n}{2}[a+l]$$

$$S_6 = \frac{6}{2}[a^2 + b^2 - 2ab + a^2 + b^2 + 2ab + 6ab]$$

$$= 6[a^2 + b^2 + 3ab]$$

A.P formed is 1, 2, 3, 4, ..., n.

Here,
$$a=1$$

$$l = n$$

So sum of
$$n$$
 terms = $S_n = \frac{n}{2} [2a + (n-1)d]$

$$= \frac{n}{2} [2 + (n-1)1]$$

$$= \frac{n(n+1)}{2}$$
 is the sum of first n natural numbers.

Arithematic Progressions Ex 19.4 Q4

The natural numbers which are divisible by 2 or 5 are:

$$2+4+5+6+8+10+\cdots+100 = (2+4+6+\cdots+100)+(5+15+25+\cdots+95)$$
 Now $(2+4+6+\cdots+100)$ and $(5+15+25+\cdots+95)$ are AP with common difference 2 and 10 respectively.

Therefore

$$2+4+6+\cdots+100 = 2\frac{50}{2}(1+50)$$
$$= 2550$$

Again

$$5+15+25+\dots+95 = 5(1+3+5+\dots+19)$$
$$= 5\left(\frac{10}{2}\right)(1+19)$$
$$= 500$$

Therefore the sum of the numbers divisible by 2 or 5 is:

$$2+4+5+6+8+10+\cdots+100 = 2550+500$$

= 3050

Arithematic Progressions Ex 19.4 Q5

The series of n odd natural numbers are 1, 3, 5, ..., n

Where n is odd natural number

Then, sum of n terms is

$$S_n = \frac{n}{2} [2a + (n-1)d]$$
$$= \frac{n}{2} [2(1) + (n-1)(2)]$$
$$= n^2$$

The sum of n odd natural numbers is n^2 .

The series so formed is 101, 103, 105, ..., 199

Let number of terms be n

Then,

$$a_n = a + (n-1)d = 199$$

$$\Rightarrow$$
 199 = 101 + $(n-1)$ 2

$$\Rightarrow$$
 $n = 50$

The sum of
$$n$$
 terms = $S_n = \frac{n}{2}[a+l]$

$$S_{50} = \frac{50}{2}[101+199]$$
= 7500

The sum of odd numbers between 100 and 200 is 7500.

Arithematic Progressions Ex 19.4 Q7

The odd numbers between 1 and 100 divisible by 3 are 3, 9, 15, ..., 999

Let the number of terms be n then, nth term is 999.

$$a_n = a(n-1)d$$

$$999 = 3 + (n - 1)6$$

$$\Rightarrow$$
 $n = 167$

The sum of n terms

$$S_n = \frac{n}{2} [a+l]$$

$$\Rightarrow$$
 $S_{167} = \frac{167}{2}[3 + 999]$
= 83667 Hence proved.

Arithematic Progressions Ex 19.4 Q8

The required series is 85, 90, 95, ..., 715

Let there be n terms in the A.P.

Then,

$$n$$
th term = 715

$$715 = 85 + (n - 1)5$$

$$n = 127$$

Then,

$$S_n = \frac{n}{2} [a+l]$$

$$S_{127} = \frac{127}{2} [85 + 715]$$

The series of integers divisble by 7 between 50 and 500 are 56, 63, 70, ..., 497

Let the number of terms be n then, nth term = 497

$$a_n = a + (n-1)d$$

 $\Rightarrow 497 = 56 + (n-1)7$

$$\Rightarrow$$
 $n = 64$

The sum
$$S_n = \frac{n}{2}[a+l]$$

$$\Rightarrow S_{64} = \frac{64}{2}[56+497]$$

$$= 32 \times 553$$

$$= 17696$$

Arithematic Progressions Ex 19.4 Q10

All even integers will have common difference = 2

$$\begin{array}{ll} \therefore & \text{A.P is } 102, 104, 106, ..., 998 \\ t_n = a + (n - 1)d \\ t_n = 998, a = 102, d = 2 \\ 998 = 102 + (n - 1)(2) \\ 998 = 102 + 2n - 2 \\ 998 - 100 = 2n \\ 2n = 898 \\ n = 449 \end{array}$$

 S_{449} can be calculated by

$$S_n = \frac{n}{2} [a+l]$$

$$= \frac{449}{2} [102 + 998]$$

$$= \frac{449}{2} \times 1100$$

$$= 449 \times 550$$

$$= 246950$$

Arithematic Progressions Ex 19.4 Q11

The series formed by all the integers between 100 and 550 which are divisible by 9 is 108,117,123,...,549

Let there be n terms in the A.P then, the nth term is 549

$$549 = a + (n - 1)d$$
$$549 = 108 + (n - 1)9$$

$$\Rightarrow$$
 $n = 50$

Then,

In the given series 3+5+7+9+... to 3n

Here,

$$a = 3$$

$$d = 2$$

Number of terms = 3n

The sum of n term is

$$S_n = \frac{n}{2} \left[2a + (n-1)d \right]$$

$$\Rightarrow S_{3n} = \frac{3n}{2} [6 + (3n - 1)2]$$
$$= 3n (2n + 3)$$

Arithematic Progressions Ex 19.4 Q13

The first number between 100 and 800 which on division by 16 leaves the remainder 7 is 112 and last number is 791.

Thus, the series so formed is 103,119,...,791

Let number of terms be n, then

$$n$$
th term = 791

Then,

$$a_n = a + (n-1)d$$

$$\Rightarrow$$
 791 = 103 + $(n-1)$ 16

Then, sum of all terms of the given series is

$$S_{43} = \frac{44}{2} [103 + 791]$$
$$= \frac{44 \times 894}{2}$$
$$= 19668$$

(i)
$$25+22+19+16+...+x = 115$$

Here, sum of the given series of say n terms is 115 So, the nth term = x

Here,
$$a = 25$$
 and $d = 22 - 25 = -3$

$$a_n = a + (n - 1)d$$

$$\Rightarrow \qquad x = 25 - 3(n - 1)$$

$$\Rightarrow \qquad x = 28 - 3n \qquad \qquad ---(i)$$

The sum of n terms

$$S_n = \frac{n}{2} [a+l]$$

$$\Rightarrow 115 = \frac{n}{2} [25 + 28 - 3n]$$

$$\Rightarrow 230 = 53n - 3n^2$$

$$\Rightarrow 3n^2 - 53n - 230 = 0$$

$$\Rightarrow 3n^2 - 30n - 23n - 230 = 0$$

$$\Rightarrow n = 10 \text{ or } \frac{23}{3}$$

But n can't be function

$$n = 10$$
 ——(ii)

$$x = 28 - 3n$$

$$X = -2$$

(ii)
$$1+4+7+10+...+x = 590$$

Here, $a=1$
 $d=4-1=3$

Let there be n terms so the nth term = x

$$\Rightarrow$$
 $x = 1 + (n-1)3$

$$\left[\because a_n = a + (n-1)d\right]$$

$$\Rightarrow x = 3n - 2$$

and

$$S_n = 590$$

$$\Rightarrow \frac{n}{2}[a+l] = 590$$

$$\Rightarrow \frac{n}{2}[1+3n-2]=590$$

$$[\because l = x = 3n - 2]$$

$$\Rightarrow 3n^2 - n - 1080 = 0$$

$$\Rightarrow$$
 $3n^2 - 60n + 59n - 1080 = 0$

$$\Rightarrow$$
 $3n(n-20)+59(n-20)=0$

$$\Rightarrow$$
 $n = 20$

$$x = 3n - 2$$

$$x = 58$$

Arithematic Progressions Ex 19.4 Q15

Sum first n terms of the given AP is

$$S_n = 3n^2 + 2n$$

$$S_{n-1} = 3(n-1)^2 + 2(n-1)$$

$$a_n = S_n - S_{n-1}$$

$$a_n = 3n^2 + 2n - 3(n-1)^2 - 2(n-1)$$

$$a_n = 6n - 1$$

$$a_{r} = 6r - 1$$

Given,

$$a_1 = -14 = a + 0d$$
 --- (i)
 $a_5 = 2 = a + 4d$ --- (ii)

Solving (i) and (ii)
$$a_1 = a = -14 \text{ and } d = 4$$

Let ther be n terms then sum of there n terms = 40

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$\Rightarrow 40 = \frac{n}{2} \left[-28 + (n-1) 4 \right]$$

$$\Rightarrow 4n^2 - 32n - 80 = 0$$

or
$$n = 10 \text{ or } -2$$

But n can't be negative

$$n = 10$$

The given A.P has 10 terms.

Arithematic Progressions Ex 19.4 Q17

Given,

$$a_7 = 10$$

$$S_{14} - S_7 = 17$$

$$S_{14} = 17 + S_7 = 17 + 10 = 27$$
---(ii)

From (i) and (ii)

⇒ and

 \Rightarrow

$$S_{14} = \frac{14}{2} [2a + 13d]$$

27 = 28a + 182d ----(iv)

Solving (iii) and (iv)

$$a = 1$$
 and $d = \frac{1}{7}$

:. The required A.P is

$$1, 1 + \frac{1}{7}, 1 + \frac{2}{7}, 1 + \frac{3}{7}, \dots, +\infty$$

or
$$1, \frac{8}{7}, \frac{9}{7}, \frac{10}{7}, \frac{11}{7}, \dots, \infty$$

$$a_3 = 7 = a + 2d$$
 ---(i)
 $a_7 = 3a_3 + 2$
 $a_7 = 3(7) + 2$ [$\because a_3 = 7$]
 $a_7 = 3 = a + 6d$ ---(ii)

solving (i) and (ii)
$$a = -1$$
, $d = 4$

Then, sum of 20 terms of this A.P

$$\Rightarrow S_{20} = \frac{20}{2} [2 + (20 - 1)4] \qquad \left[\text{Using } S_n = \frac{n}{2} [2a + (n - 1)d] \right]$$

$$= 10 \times 74$$

$$= 740$$

First term is -1 common defference = 4, sum of 20 terms = 740.

Arithematic Progressions Ex 19.4 Q19

Given,

$$a = 2$$
 $l = 50$

$$\therefore l = a + (n - 1)d$$

$$50 = 2 + (n - 1)d$$

$$(n - 1)d = 48$$
---(i)

 S_n of all n terms is given 442

$$S_n = \frac{n}{2} [a+l]$$

$$442 = \frac{n}{2} [2+50]$$
or $n = 17$ ---- (ii)

From (i) and (ii)
$$d = \frac{48}{p-1} = \frac{48}{16} = 3$$

The common difference is 3.

Let no. of terms be 2n

Odd terms sum= $24=T_1+T_3+...+T_{2n-1}$

Even terms sum= $30=T_2+T_4+...+T_{2n}$

Subtract above two equations

nd=6

$$T_{2n} = T_1 + \frac{21}{2}$$

$$T_{2n} - \alpha = \frac{21}{2}$$

$$(2n-1)d = \frac{21}{2}$$

$$12 - \frac{21}{2} = d = \frac{3}{2}$$

$$\Rightarrow n = 6 \times \frac{2}{3} = 4$$

Total terms = 2n = 8

Subtitute above values in equation of sum of even terms or odd terms, we get

$$a=\frac{3}{2}$$

So series is $\frac{3}{2}$, 3, $\frac{9}{2}$

Let a be the first term of the AP and d is the common difference. Then

$$\begin{split} S_n &= \frac{n}{2} \Big(2a + (n-1)d \Big) \\ n^2 p &= \frac{n}{2} \Big(2a + (n-1)d \Big) \\ np &= \frac{1}{2} \Big[2a + (n-1)d \Big] \\ 2np &= 2a + (n-1)d \qquad(1) \end{split}$$

Again

$$S_{m} = \frac{m}{2} (2a + (m-1)d)$$

$$m^{2} p = \frac{m}{2} (2a + (m-1)d)$$

$$mp = \frac{1}{2} [2a + (m-1)d]$$

$$2mp = 2a + (m-1)d \qquad(2)$$
Now subtract (1) from (2)
$$2p(m-n) = (m-n)d$$

Therefore

$$2mp = 2a + (m-1) \cdot 2p$$
$$2a = 2p$$
$$a = p$$

d = 2p

The sum up to p terms will be:

$$S_{p} = \frac{p}{2} (2a + (p-1)d)$$

$$= \frac{p}{2} (2p + (p-1) \cdot 2p)$$

$$= \frac{p}{2} (2p + 2p^{2} - 2p)$$

$$= p^{3}$$

Hence it is shown.

Arithematic Progressions Ex 19.4 Q22

$$a_{12} = a + 11d = -13 \qquad ---(i) \qquad \text{[Given]}$$

$$s_4 = \frac{4}{2}(2a + 3d) = 24 \qquad ---(ii) \qquad \text{[Given]}$$
From (i) and (ii)
$$d = -2 \text{ and } a = 9$$
Then,
$$\text{Sum of irst 10 terms is}$$

$$S_{10} = \frac{10}{2}[2 \times 9 + (9)(-2)] \qquad \qquad \left[\text{Using } S_n = \frac{n}{2}[2a + (n-1)d] \right]$$

Sum of first 10 terms is zero.

$$a_5 = a + 4d = 30$$
 ---(i) [Given]
 $a_{12} = a + 11d = 65$ ---(ii) [Given]

Then,

Sum of irst 20 terms is

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$S_{20} = \frac{20}{2} [2 \times 10 + (20-1)5]$$

$$= 1150$$

Sum of first 20 terms is 1150.

Arithematic Progressions Ex 19.4 Q24

Here,

$$a_k = 5k + 1$$

$$a_1 = 5 + 1 = 6$$

$$a_2 = 5(2) + 1 = 11$$

$$a_3 = 5(3) + 1 = 16$$

$$d = 11 - 6 = 16 - 11 = 5$$

$$S_n = \frac{n}{2} [2a + (n - 1)d]$$

$$= \frac{n}{2} [2(6) + (n - 1)(5)]$$

$$= \frac{n}{2} [12 + 5n - 5]$$

$$S_n = \frac{n}{2} (5n + 7)$$

Arithematic Progressions Ex 19.4 Q25

sum of all two digit numbers which when divided by 4, yields 1 as remainder, \Rightarrow all 4n+1 terms with n \geq 3 13,17,21,..............97 n=22, a=13, d=4 sum of terms = $\frac{22}{2}[26+21\times4]=11\times110=1210$

Sum of terms 25, 22, 19,...., is 116

$$\frac{n}{2} [50 + (n-1)(-3)] = 116$$

$$\frac{n}{2}[53-3n]=116$$

$$53n - 3n^2 = 232$$

$$3n^2 - 53n + 232 = 0$$

$$3n^2 - 29n - 24n + 232 = 0$$

$$n(3n-29)-8(3n-29)=0$$

$$(3n-29)(n-8)=0$$

$$\Rightarrow n = 8or \frac{29}{3}$$

n cannot be in fraction, so n=8

last term= $25-7\times3=4$

Arithematic Progressions Ex 19.4 Q27

Let the number of terms is n.

Now the sum of the series is:

$$1+3+5+\cdots+2001$$

Here l = 2001 and d = 2.

Therefore

$$l = a + (n-1)d$$

$$2001 = 1 + (n-1) \cdot 2$$

$$2(n-1) = 2000$$

$$n-1=1000$$

$$n = 1001$$

Therefore the sum of the series is:

$$S = \frac{1001}{2} [2 + (1001 - 1)2]$$
$$= 1001^{2}$$
$$= 1002001$$

Arithematic Progressions Ex 19.4 Q28

Let the number of terms to be added to the series is n.

Now
$$a = -6$$
 and $d = 0.5$.

Therefore

$$-25 = \frac{n}{2} \Big[2(-6) + (n-1)(0.5) \Big]$$

$$-50 = n \Big[-12 + 0.5n - 0.5 \Big]$$

$$-12.5n + 0.5n^2 + 50 = 0$$

$$n^2 - 25n + 100 = 0$$

$$n = 20.5$$

Therefore the value of n will be either 20 or 5.

Here the first term a=2. Let the common difference is d.

Now

$$\frac{5}{2} \left[2a + (5-1)d \right] = \frac{1}{4} \left[\frac{5}{2} \left[2(a+5d) + (5-1)d \right] \right]$$

$$\frac{5}{2} \left[2 \cdot 2 + 4d \right] = \frac{5}{8} \left[2 \cdot 2 + 14d \right]$$

$$10 + 10d = \frac{5}{2} + \frac{35}{4}d$$

$$\frac{5}{4}d = -7.5$$

The 20th term will be:

$$a + (n-1)d = 2 + (20-1)(-6)$$
$$= -112$$

Hence it is shown.

Arithematic Progressions Ex 19.4 Q30

$$\begin{split} S_{(2n+1)} &= S_1 = \frac{(2n+1)}{2} \big[2a + (2n+1-1)d \big] \\ S_1 &= \frac{(2n+1)}{2} \big[2a + 2nd \big] \\ &= (2n+1) \big(a + nd \big) \end{split} \qquad \qquad --- (i) \\ \text{Sum of odd term} \, s &= S_2 \end{split}$$

$$\begin{split} S_2 &= \frac{(n+1)}{2} \Big[2a + (n+1-1)(2d) \Big] \\ &= \frac{(n+1)}{2} \Big[2a + 2nd \Big] \\ S_2 &= (n+1)(a+nd) \end{split} \qquad \qquad ---(ii) \end{split}$$

From equation (i) and (ii),

$$S_1: S_2 = (2n+1)(a+nd): (n+1)(a+nd)$$

 $S_1: S_2 = (2n+1); (n+1)$

Here,

$$S_n = 3n^2$$

[Given]

Where n is number of term

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

---(ii)

From (i) and (ii)

$$3n^2 = \frac{n}{2} \left[2a + (n-1)d \right]$$

$$6n = 2a + nd - d$$

Equating both sides

$$6n = nd$$

$$d = 6$$

and

$$0 = 2a - d$$

or
$$d = 2a$$

From (iii) and (iv)

$$a = 3$$
 and $d = 6$

∴ The required A.P is 3, 9, 15, 21, ..., ∞

Arithematic Progressions Ex 19.4 Q32

$$S_n = nP + \frac{1}{2}n(n-1)Q$$

[Given]

$$S_n = \frac{n}{2} \left[2P + (n-1)Q \right]$$

---(i)

We know

$$S_n = \frac{n}{2} \left[2a + (n-1)d \right]$$

Where a =first term and d =common difference comparing (i) and (ii) d = Q

: The common difference is Q.

Let sum of n terms of two A.P be S_n and S'n.

Then, $S_n = 5n + 4$ and $S'_n = 9n + 16$ respectively.

Then, if ratio of sum of n terms of 2A.P is giben, then the ratio of there nth ther is obtained by replacing n by (2n-1).

$$\frac{a_n}{a_n'} = \frac{5(2n-1)+4}{9(2n-1)+16}$$

3. Ratio of there 18th term is

$$\frac{a_{18}}{a'_{18}} = \frac{5(2 \times 18 - 1) + 4}{9(2 \times 18 - 1) + 16}$$
$$= \frac{5 \times 35 + 4}{9 \times 35 + 16}$$
$$= \frac{179}{321}$$

Arithematic Progressions Ex 19.4 Q34

Let sum of n term of 1 A.P series be S_n are other S_n

The,
$$S_n = 7n + 2$$
 ---(i). $S_n = n + 4$ ---(ii)

If the ratio of sum of n terms of 2 A.P is given, then the ratio of there nth term is obtained by replacing n by (2n-1).

$$\frac{a_n}{a_{n'}} = \frac{7(2n-1)+2}{(2n-1)+4}$$

Putting n = 5 to get the ratio of 5th term, we get

$$\frac{a_5}{a'5} = \frac{7(2 \times 5 - 1) + 2}{(2 \times 5 - 1) + 4} = \frac{65}{13} = \frac{5}{1}$$

The ratio is 5 : 1.