RD Sharma Solutions Class 11 Maths Chapter 18 Ex 18.1

Binomial Theorem Ex 18.1 Q1(i)

The expansion of $(x+y)^n$ has n+1 term so, the expansion of $(2x+3y)^5$ has 6 terms.

Using binomial theorem, we have

$$(2x+3y)^5 = {}^5C_0(2x)^5(3y)^0 + {}^5C_1(2x)^4(3y)^1 + {}^5C_2(2x)^3(3y)^2 + {}^5C_3(2x)^2(3y)^3 + {}^5C_4(2x)(3y)^4 + {}^5C_5(2x)^0(3y)^5$$

$$= 2^5x^5 + 5\times 2^4\times 3\times x^4\times y + 10\times 2^3\times 3^2\times x^3\times y^2 + 10\times 2^2\times 3^3\times x^2\times y^3 + 5\times 2\times 3^4\times x\times y^4 + 3^5y^5$$

$$= 32x^5 + 240x^4y + 720x^3y^2 + 1080x^2y^3 + 810xy^4 + 243y^5$$

Binomial Theorem Ex 18.1 Q1(ii)

The expansion of $(x+y)^n$ has $n+1$ terms so the expansion of $(2x-3y)^4$ has 5 terms.

Using binomial theorem, we have

$$(2x-3y)^4 = {}^4C_0(2x)^4(3y)^0 - {}^4C_1(2x)^3(3y)^1 + {}^4C_2(2x)^2(3y)^2 - {}^4C_3(2x)^1(3y)^3 + {}^4C_4(2x)^0(3y)^4$$

$$= 2^4x^4 - 4\times 2^3\times 3x^3y + 6\times 2^2\times 3^2\times x^2y^2 - 4\times 2\times 3^3\times xy^3 + 3^4y^4$$

$$= 16x^4 - 96x^3y + 216x^2y^2 - 216xy^3 + 81y^4$$

Binomial Theorem Ex 18.1 Q1(iii)

The expansion of $(x+y)^n$ has $n+1$ terms so the expansion of $\left(x-\frac{1}{x}\right)^6$ has 7 term.

Using binomial theorem, we get

$$\left(x-\frac{1}{x}\right)^6 = {}^6C_0x^6\left(\frac{1}{x}\right)^0 - {}^6C_1x^5\left(\frac{1}{x}\right) + {}^6C_2x^4\left(\frac{1}{x}\right)^2 - {}^6C_3x^3\left(\frac{1}{x}\right)^3 + {}^6C_4x^2\left(\frac{1}{x}\right)^4 - {}^6C_5x\left(\frac{1}{x}\right)^5 + {}^6C_6x^0\left(\frac{1}{x}\right)^6$$

$$= x^6 - 6x^4 + 15x^2 - 20 + \frac{15}{x^2} - \frac{6}{x^4} + \frac{1}{x^6}$$

Binomial Theorem Ex 18.1 Q1(iv)

The expansion of $(x+y)^n$ has $n+1$ terms so the expansion of $(1-3x)^7$ has 8 term.

Using binomial theorem to expand, we get

$$(1-3x)^7 = {}^7C_0(1)^7(3x)^0 - {}^7C_1(3x) + {}^7C_2(3x)^2 - {}^7C_3(3x)^3 + {}^7C_4(3x)^4 - {}^7C_5(3x)^5 - {}^7C_6(3x)^6 + {}^7C_7(3x)^7$$

$$= 1 - 21x + 21 \times 9x^2 - 35 \times 3^3x^3 + 35 \times 3^4x^4 - 21 \times 3^5x^5 + 7 \times 3^6x^6 - 3^7x^7$$

$$= 1 - 21x + 189x^2 - 945x^3 + 2835x^4 - 5103x^5 + 5103x^6 - 2187x^7$$

Binomial Theorem Ex 18.1 Q1(v)

The expansion of $(x+y)^n$ has $n+1$ terms so the expansion of $\left(ax-\frac{b}{x}\right)^6$ has 7 terms.

Using binomial theorem to expand, we get

$$\left(ax-\frac{b}{x}\right)^6 = {}^6C_0(ax)^6\left(\frac{b}{x}\right)^0 - {}^6C_1(ax)^5\left(\frac{b}{x}\right) + {}^6C_2(ax)^4\left(\frac{b}{x}\right)^2 - {}^6C_3(ax)^3\left(\frac{b}{x}\right)^3 + {}^6C_4(ax)^2\left(\frac{b}{x}\right)^4 - {}^6C_5(ax)\left(\frac{b}{x}\right)^5 + {}^6C_6(ax)^0\left(\frac{b}{x}\right)^6$$

$$= a^6x^6 - 6a^5x^5\frac{b}{x} + 15a^4x^4\frac{b^2}{x^2} - 20a^3b^3 + 15a^2\frac{b^4}{x^2} - 6a\frac{b^5}{x^4} + \frac{b^6}{x^6}$$

$$= a^6x^6 - 6a^5x^4b + 15a^4b^2x^2 - 20a^3b^3 + 15\frac{a^2b^4}{x^2} - 6\frac{ab^5}{x^4} + \frac{b^6}{x^6}$$

Binomial Theorem Ex 18.1 Q1(vi)

The expansion of $(x+y)^n$ has $n+1$ terms so the expansion of $\left(\sqrt{\frac{x}{a}}-\sqrt{\frac{a}{x}}\right)^6$ has 7 terms.

Using binomial theorem to expand, we get

$$\left(\sqrt{\frac{x}{a}}-\sqrt{\frac{a}{x}}\right)^6 = {}^6C_0\left(\sqrt{\frac{x}{a}}\right)^6\left(\sqrt{\frac{a}{x}}\right)^0 - {}^6C_1\left(\sqrt{\frac{x}{a}}\right)^5\left(\sqrt{\frac{a}{x}}\right)^1 + {}^6C_2\left(\sqrt{\frac{x}{a}}\right)^4\left(\sqrt{\frac{a}{x}}\right)^2 - {}^6C_3\left(\sqrt{\frac{x}{a}}\right)^3\left(\sqrt{\frac{a}{x}}\right)^3$$

$$+{}^6C_4\left(\sqrt{\frac{x}{a}}\right)^2\left(\sqrt{\frac{a}{x}}\right)^4 - {}^6C_5\left(\sqrt{\frac{x}{a}}\right)\left(\sqrt{\frac{a}{x}}\right)^5 + {}^6C_6\left(\sqrt{\frac{x}{a}}\right)^0\left(\sqrt{\frac{a}{x}}\right)^6$$

$$=\left(\frac{x}{a}\right)^{\frac{1}{2}\times 6} - 6\left(\frac{x}{a}\right)^{\frac{1}{2}\times 5}\left(\frac{a}{x}\right)^{\frac{1}{2}} + 15\left(\frac{x}{a}\right)^{\frac{1}{2}\times 4}\left(\frac{a}{x}\right)^{2\times\frac{1}{2}} - 20\left(\frac{x}{a}\right)^{3\times\frac{1}{2}}\left(\frac{a}{x}\right)^{3\times\frac{1}{2}} + 15\left(\frac{x}{a}\right)^{2\times\frac{1}{2}}\left(\frac{a}{x}\right)^{4\times\frac{1}{2}}$$

$$-6\left(\frac{x}{a}\right)^{\frac{1}{2}}\left(\frac{a}{x}\right)^{5\times\frac{1}{2}} + \left(\frac{a}{x}\right)^{6\times\frac{1}{2}}$$

$$=\frac{x^3}{a^3} - 6\frac{x^{\frac{5}{2}-\frac{1}{2}}}{a^{\frac{5}{2}-\frac{1}{2}}} + 15\times\frac{x^2\times a}{a^2\times x} - 20\times\frac{x^{\frac{3}{2}-\frac{3}{2}}}{a^{\frac{3}{2}-\frac{3}{2}}} + 15\times\frac{x}{a}\times\frac{a^2}{x^2} - 6\times\frac{x^{\frac{1}{2}-\frac{5}{2}}}{a^{\frac{1}{2}-\frac{5}{2}}} + \frac{a^3}{x^3}$$

$$=\frac{x^3}{a^3} - \frac{6x^2}{a^2} + \frac{15x}{a} - 20 + \frac{15a}{x} - \frac{6a^2}{x^2} + \frac{a^3}{x^3}$$

Binomial Theorem Ex 18.1 Q1(vii)

$$\left(\sqrt[3]{x}-\sqrt[3]{a}\right)^6$$

$$=\binom{6}{0}\left(\sqrt[3]{x}\right)^6\left(-\sqrt[3]{a}\right)^0 + \binom{6}{1}\left(\sqrt[3]{x}\right)^5\left(-\sqrt[3]{a}\right)^1 + \binom{6}{2}\left(\sqrt[3]{x}\right)^4\left(-\sqrt[3]{a}\right)^2$$

$$\binom{6}{3}\left(\sqrt[3]{x}\right)^3\left(-\sqrt[3]{a}\right)^3 + \binom{6}{4}\left(\sqrt[3]{x}\right)^2\left(-\sqrt[3]{a}\right)^4 + \binom{6}{5}\left(\sqrt[3]{x}\right)^1\left(-\sqrt[3]{a}\right)^5$$

$$\binom{6}{6}\left(\sqrt[3]{x}\right)^0\left(-\sqrt[3]{a}\right)^6$$

$$= x^2 - 6x^{\frac{5}{3}}a^{\frac{1}{3}} + 15x^{\frac{4}{3}}a^{\frac{2}{3}} - 20ax + 15x^{\frac{2}{3}}a^{\frac{4}{3}} - 6x^{\frac{1}{3}}a^{\frac{5}{3}} + a^2$$

Binomial Theorem Ex 18.1 Q1(viii)

Let $y = 1+2x$, then

$$\left(1+2x-3x^2\right)^5 = \left(y-3x^2\right)^5$$

The expansion of $(x+y)^n$ has $n+1$ terms so the expansion of $\left(y-3x^2\right)^5$ has 6 terms.

Using binomial theorem to expand, we get

$$\left(y-3x^2\right)^5 = {}^5C_0 y^5\left(3x^2\right)^0 - {}^5C_1 y^4\left(3x^2\right)^1 + {}^5C_2 y^3\left(3x^2\right)^2 - {}^5C_3 y^2\left(3x^2\right)^3 + {}^5C_4 y\left(3x^2\right)^4 - {}^5C_5 y^0\left(3x^2\right)^5$$
$$= y^5 - 5y^4\ 3x^2\ + 10y^3\ 9x\ ^4 - 10y^2\left(27x^6\right) + 5y81x^8 - 243x^{10}$$

Now,

$$y^5 = (1+2x)^5 = {}^5C_0 + {}^5C_1(2x)^1 + {}^5C_2(2x)^2 + {}^5C_3(2x)^3 + {}^5C_4(2x)^4 + {}^5C_5(2x)^5$$
$$y^4 = (1+2x)^4 = {}^4C_0 + {}^4C_1(2x)^1 + {}^4C_2(2x)^2 + {}^4C_3(2x)^3 + {}^4C_4(2x)^4$$
$$y^3 = (1+2x)^3 = {}^3C_0 + {}^3C_1(2x) + {}^3C_2(2x)^2 + {}^3C_3(2x)^3$$
$$y^2 = (1+2x)^2 = {}^2C_0 + {}^2C_1(2x) + {}^2C_2(2x)^2$$
$$y = (1+2x)$$

Substituting the valus of powers of y in the equation above, we get,

$$\left(1+2x-3x^2\right)^5 = \left[{}^5C_0 + {}^5C_1(2x)^1 + {}^5C_2(2x)^2 + {}^5C_3(2x)^3 + {}^5C_4(2x)^4 + {}^5C_5(2x)^5\right]$$
$$-15x^2\left[{}^4C_0 + {}^4C_1(2x)^1 + {}^4C_2(2x)^2 + {}^4C_3(2x)^3 + {}^4C_4(2x)^4\right]$$
$$+90x^4\left[{}^3C_0 + {}^3C_1(2x) + {}^3C_2(2x)^2 + {}^3C_3(2x)^3\right] - 270x^6$$
$$\left[{}^2C_0 + {}^2C_1(2x) + {}^2C_2(2x)^2 + 5\times 81x^8(1+2x) - 243x^{10}\right]$$

$$= \quad 10 + 10x + 10\times 4x^2 + 10\times 8x^3 + 5\times 16x^4 + 32x^5 - 15x^2 - 120x^3$$
$$-180x^4 + 480x^5 - 240x^6 + 90x^4 + 540x^5 + 1080x^6 + 720x^7 - 270x^6$$
$$-1080x^7 - 1080x^8 + 405x^8 + 810x^9 - 243x^{10}$$

$$= 1 + 10x + 25x^2 - 40x^3 - 190x^4 + 92x^5 + 570x^6 - 360x^7 - 675x^8 + 810x^9 - 243x^{10}$$

Binomial Theorem Ex 18.1 Q1(ix)

Let $y = x+1$, then

$$\left(x+1-\frac{1}{x}\right)^3 = \left(y-\frac{1}{x}\right)^3$$

The expansion of $(x+y)^n$ has $n+1$ terms so the expansion of $\left(y-\frac{1}{x}\right)^3$ has 4 terms.

Using binomial theorem to expand, we get

$$\left(y-\frac{1}{x}\right)^3 = {}^3C_0 y^3\left(\frac{1}{x}\right)^0 - {}^3C_1 y^2\left(\frac{1}{x}\right) + {}^3C_2 y\left(\frac{1}{x}\right)^2 - {}^3C_3 y^0\left(\frac{1}{x}\right)^3$$

$$= y^3 - 3y^2 \times \frac{1}{x} + 3y \times \frac{1}{x^2} - \frac{1}{x^3}$$

Putting $y = x+1$, we get

$$\left(x+1-\frac{1}{x}\right)^3 = (x+1)^3 - 3(x+1)^2 \times \frac{1}{x} + 3(x+1) \times \frac{1}{x^2} - \frac{1}{x^3}$$

$$= x^3 + 1 + 3x^2 + 3x - 3x - \frac{3}{x} - 6 + \frac{3}{x} + \frac{3}{x^2} - \frac{1}{x^3}$$

$$= x^3 + 3x^2 - 5 + \frac{3}{x^2} - \frac{1}{x^3}$$

Binomial Theorem Ex 18.1 Q1(x)

Let $y = 1-2x$, then

$$\left(1-2x+3x^2\right)^3 = \left(y+3x^2\right)^3$$

The expansion of $(x+y)^n$ has $n+1$ terms so the expansion of $\left(y+3x^2\right)^3$ has 4 terms.

Using binomial theorem to expand, we get

$$\left(y+3x^2\right)^3 = {}^3C_0 y^3\left(3x^2\right)^0 + {}^3C_1 y^2\left(3x^2\right)^1 + {}^3C_2 y\left(3x^2\right)^2 + {}^3C_3 y^0\left(3x^2\right)^3$$

$$= y^3 + 3y^2\left(3x^2\right) + 3y\left(9x^2\right) + \left(27x^6\right)$$

Substituting $y = 1-2x$, we get,

$$\left(1-2x+3x^2\right)^3 = (1-2x)^3 + 3\left(1+4x^2-4x\right)\left(3x^2\right) + 3(1-2x)\left(9x^2\right) + \left(27x^6\right)$$

$$= 1 - 8x^3 - 6x + 12x^2 + 9x^2 + 36x^4 - 36x^3 + 27x^2 - 54x^3 + 27x^6$$

$$= 1 - 6x + 21x^2 - 44x^3 + 63x^4 - 54x^5 + 27x^6$$

Binomial Theorem Ex 18.1 Q2(i)

$$\left(\sqrt{x+1}+\sqrt{x-1}\right)^6+\left(\sqrt{x+1}-\sqrt{x-1}\right)^6$$

$$= {}^6C_0\left(\sqrt{x+1}\right)^6+{}^6C_1\left(\sqrt{x+1}\right)^5\left(\sqrt{x-1}\right)+{}^6C_2\left(\sqrt{x+1}\right)^4\left(\sqrt{x-1}\right)^2-{}^6C_3\left(\sqrt{x+1}\right)^3\left(\sqrt{x-1}\right)^3$$
$$+{}^6C_4\left(\sqrt{x+1}\right)^2\left(\sqrt{x-1}\right)^4+{}^6C_5\left(\sqrt{x+1}\right)\left(\sqrt{x-1}\right)^5+{}^6C_6\left(\sqrt{x-1}\right)^6+{}^6C_0\left(\sqrt{x+1}\right)^6-$$
$${}^6C_1\left(\sqrt{x+1}\right)^5\left(\sqrt{x-1}\right)+{}^6C_2\left(\sqrt{x+1}\right)^4 x\left(\sqrt{x-1}\right)^2-{}^6C_3\left(\sqrt{x+1}\right)^3\left(\sqrt{x-1}\right)^3+$$
$${}^6C_4\left(\sqrt{x+1}\right)^2\left(\sqrt{x-1}\right)^4-{}^6C_5\left(\sqrt{x+1}\right)\left(\sqrt{x-1}\right)^5+{}^6C_6\left(\sqrt{x-1}\right)^6$$

$$=2\left[(x+1)^3+15(x+1)^2(x-1)+15(x+1)(x-1)^2+(x-1)^3\right]$$

$$=2\left[\begin{matrix}x^3+1+3x+3x^2+15x^3-15x^2+15x-15+30x^2-30x\\ +15x^3+15x^2+15x+15-30x^2-30x+x^3-1-3x^2+3x\end{matrix}\right]$$

$$=64x^3-48x$$
$$=16x\left(4x^2-3\right)$$

Binomial Theorem Ex 18.1 Q2(ii)

$$\left(x+\sqrt{x^2-1}\right)^6+\left(x-\sqrt{x^2-1}\right)^6$$

$$=2\left[{}^6C_0x^6+{}^6C_2x^4\left(\sqrt{x^2-1}\right)^2+{}^6C_4x^2\left(\sqrt{x^2-1}\right)^4+{}^6C_6\left(\sqrt{x^2-1}\right)^6\right]$$

$$=2\left[x^6+15x^4\left(x^2-1\right)+15x^2\left(x^2-1\right)^2+\left(x^2-1\right)^3\right]$$

$$=2\left[x^6+15x^6-15x^4+15x^6+15x^2-30x^4+x^6-1-3x^4+3x^2\right]$$

$$=64x^6-96x^4+36x^2-2$$

Binomial Theorem Ex 18.1 Q2(iii)

$$\left(1+2\sqrt{x}\right)^5+\left(1-2\sqrt{x}\right)^5$$

$$=2\left[{}^5C_0+{}^5C_2\left(2\sqrt{x}\right)^2+{}^5C_4\left(2\sqrt{x}\right)^4\right]$$

$$\left(\sqrt{2}+1\right)^6+\left(\sqrt{2}-1\right)^6$$

$$= {}^6C_0\left(\sqrt{2}\right)^6+{}^6C_1\left(\sqrt{2}\right)^5+{}^6C_2\left(\sqrt{2}\right)^4+{}^6C_3\left(\sqrt{2}\right)^3+{}^6C_4\left(\sqrt{2}\right)^2+{}^6C_5\left(\sqrt{2}\right)+{}^6C_6+{}^6C_0\left(\sqrt{2}\right)^6-$$
$${}^6C_1\left(\sqrt{2}\right)^5+{}^6C_2\left(\sqrt{2}\right)^4-{}^6C_3\left(\sqrt{2}\right)^3+{}^6C_4\left(\sqrt{2}\right)^2-{}^6C_5\left(\sqrt{2}\right)+{}^6C_6\left(\sqrt{2}\right)^0$$

$$=2\left[2^3+15\times2^2+15\times2+1\right]$$

$$=2\left[8+60+30+1\right]=2(99)=198$$

Binomial Theorem Ex 18.1 Q2(v)

$$\left(3+\sqrt{2}\right)^5-\left(3-\sqrt{2}\right)^5$$

$$=2\left[{}^5C_1(3)^4\left(\sqrt{2}\right)^1+{}^5C_3(3)^2\left(\sqrt{2}\right)^3+{}^5C_5\left(\sqrt{2}\right)^5\right]$$

$$=2\left[5\times81\times\sqrt{2}+10\times9\times2\sqrt{2}+4\sqrt{2}\right]$$

$$=2\left[405\sqrt{2}+180\sqrt{2}+4\sqrt{2}\right]$$

$$=2\left[589\sqrt{2}\right]$$
$$=1178\sqrt{2}$$

Binomial Theorem Ex 18.1 Q2(vi)

$$\left(2+\sqrt{3}\right)^7+\left(2-\sqrt{3}\right)^7$$

$$=2\left[{}^7C_0 2^7+{}^7C_2 2^5\left(\sqrt{3}\right)^2+{}^7C_4(2)^4\left(\sqrt{3}\right)^4+{}^7C_6 2\left(\sqrt{3}\right)^6\right]$$

$$=2\left[128+21\times32\times3+35\times8\times9+7\times2\times27\right]$$

$$=2\left[128+2016+2520+378\right]$$

$$=2\left[5042\right]$$

$$=10084$$

Binomial Theorem Ex 18.1 Q2(vii)

$$\left(\sqrt{3}+1\right)^5-\left(\sqrt{3}-1\right)^5$$

$$=2\left[{}^5C_1\left(\sqrt{3}\right)^4+{}^5C_3\left(\sqrt{3}\right)^2+{}^5C_5\right]$$

$$=2\left[5\times9+10\times3+1\right]$$

$$=2\left[45+30+1\right]$$

$$=2\left[76\right]$$
$$=152$$

Binomial Theorem Ex 18.1 Q2(viii)

$$(0.99)^5+(1.01)^5$$

$$=(1-.01)^5+(1+.01)^5$$
$$=2\left[{}^5C_1+{}^5C_3(.01)^2+{}^5C_5(.01)^5\right]$$

$$=2\left[5+10\times\frac{1}{10^4}+\frac{1}{10^{10}}\right]$$

$$=2\left[5+\frac{1}{1000}+\frac{1}{10^{10}}\right]$$

$$=2.0020001$$

Binomial Theorem Ex 18.1 Q2(ix)

$$\left(\sqrt{3}+\sqrt{2}\right)^6-\left(\sqrt{3}-\sqrt{2}\right)^6$$

$$=2\left[{}^6C_1\left(\sqrt{3}\right)^5\left(\sqrt{2}\right)+{}^6C_3\left(\sqrt{3}\right)^3\left(\sqrt{2}\right)^3+{}^6C_5\left(\sqrt{3}\right)\left(\sqrt{2}\right)^5\right]$$

$$=2\left[6\times\sqrt{6}\times9+20\times3\sqrt{3}\times2\sqrt{2}+6\times\sqrt{3}\times4\sqrt{2}\right]$$

$$=2\left[54\sqrt{6}+120\sqrt{6}+24\sqrt{6}\right]$$

$$=2\left[198\sqrt{6}\right]$$
$$=396\sqrt{6}$$

Binomial Theorem Ex 18.1 Q2(x)

$$\left\{a^2+\sqrt{a^2-1}\right\}^4+\left\{a^2-\sqrt{a^2-1}\right\}^4$$

Let $a^2 = A, \quad \sqrt{a^2-1} = B$

$$(A+B)^4+(A-B)^4$$
$$= B^4 + {}^4C_1AB^3 + {}^4C_2A^2B^2 + {}^4C_3A^3B + A^4 + B^4 - {}^4C_1AB^3 + {}^4{}_2A^2B^2 - {}^4C_3A^3B + A^4$$
$$= 2\left(A^4 + {}^4C_2A^2B^2 + B^4\right)$$
$$= 2\left(A^2 + 6A^2B^2 + B^4\right)$$
$$= 2\left(a^8 + 6a^4\left(a^2-1\right) + \left(a^2-1\right)^2\right)$$
$$= 2\left[a^8 + 6a^6 - 6a^4 + a^4 + 1 - 2a^2\right]$$

$$\left\{a^2+\sqrt{a^2-1}\right\}^4+\left\{a^2-\sqrt{a^2-1}\right\}^4 = 2a^8 + 12a^6 - 10a^4 - 4a^4 + 2$$

Binomial Theorem Ex 18.1 Q3

We have,

$$(a+b)^4-(a-b)^4$$
$$= \left[{}^4C_0a^4b^0 + {}^4C_1a^3b^1 + {}^4C_2a^2b^2 + {}^4C_3a^1b^3 + {}^4C_4a^0b^4\right]$$
$$-\left[{}^4C_0a^4b^0 - {}^4C_1a^3b^1 + {}^4C_2a^2b^2 - {}^4C_3a^1b^3 + {}^4C_4a^0b^4\right]$$
$$= \left[{}^4C_0a^4(-b)^0 + {}^4C_1a^3(-b)^1 + {}^4C_2a^2(-b)^2 + {}^4C_3a^1(-b)^3 + {}^4C_4a^0(-b)^4\right]$$
$$-\left[{}^4C_0a^4(-b)^0 + {}^4C_1a^3(-b)^1 + {}^4C_2a^2(-b)^2 + {}^4C_3a^1(-b)^3 + {}^4C_4a^0(-b)^4\right]$$
$$= \left[{}^4C_0a^4 + {}^4C_1a^3b + {}^4C_2a^2b^2 + {}^4C_3ab^3 + {}^4C_4ab^4\right] - \left[{}^4C_0a^4 - {}^4C_1a^3b + {}^4C_2a^2b^2 - {}^4C_3ab^3 + {}^4C_4b^4\right]$$
$$= {}^4C_0a^4 + {}^4C_1a^3b + {}^4C_2a^2b^2 + {}^4C_3ab^3 + {}^4C_4ab^4 - {}^4C_0a^4 + {}^4C_1a^3b - {}^4C_2a^2b^2 + {}^4C_3ab^3 - {}^4C_4b^4$$
$$= 2\left[{}^4C_1a^3b + {}^4C_3ab^3\right]$$
$$= 2\left[4a^3b + 4ab^3\right]$$
$$= 8\left[a^3b + ab^3\right]$$

$\therefore \quad (a+b)^4-(a-b)^4 = 8\left(a^3b+ab^3\right) \quad$ ---(i)

Putting $a = \sqrt{3}$ and $b = \sqrt{2}$ in equation (i), we get

$$\left(\sqrt{3}+\sqrt{2}\right)^4 - \left(\sqrt{3}-\sqrt{2}\right)^4 = 8\left[\left(\sqrt{3}\right)^3 \times \sqrt{2} + \left(\sqrt{3}\right) \times \left(\sqrt{2}\right)^3\right]$$
$$= 8\left[3\sqrt{6} + 2\sqrt{6}\right]$$
$$= 8 \times 5\sqrt{6}$$
$$= 40\sqrt{6}$$

$\therefore \quad \left(\sqrt{3}+\sqrt{2}\right)^4 - \left(\sqrt{3}-\sqrt{2}\right)^4 = 40\sqrt{6}.$

Binomial Theorem Ex 18.1 Q4

We have,

$$(x+1)^6 - (x-1)^6$$
$$= \left[{}^6C_0x^6 + {}^6C_1x^5 + {}^6C_2x^4 + {}^6C_3x^3 + {}^6C_4x^2 + {}^6C_5x^1 + {}^6C_6x^0\right]$$
$$+\left[{}^6C_0x^6(-1)^0 + {}^6C_1x^5(-1)^1 + {}^6C_2x^4(-1)^2 + {}^6C_3x^3(-1)^3 + {}^6C_4x^2(-1)^4 + {}^6C_5x^1(-1)^5 + {}^6C_6x^0(-1)^6\right]$$
$$= \left[{}^6C_0x^6 + {}^6C_1x^5 + {}^6C_2x^4 + {}^6C_3x^3 + {}^6C_4x^2 + {}^6C_5x + {}^6C_6 + {}^6C_0x^6 - {}^6C_1x^5 + {}^6C_2x^4 - {}^6C_3x^3 + {}^6C_4x^2 - {}^6C_5x + {}^6C_6\right]$$
$$= 2\left[{}^6C_0x^6 + {}^6C_2x^4 + {}^6C_4x^2 + {}^6C_6\right]$$
$$= 2\left[x^6 + 15x^4 + 15x^2 + 1\right]$$

$$\therefore \quad (x+1)^6 + (x-1)^6 = 2\left[x^6 + 15x^4 + 15x^2 + 1\right] \qquad \text{---(i)}$$

Putting $x = \sqrt{2}$ in equation (i), we get

$$(x+1)^6 + (x-1)^6 = 2\left[\left(\sqrt{2}\right)^6 + 15\left(\sqrt{2}\right)^4 + 15\left(\sqrt{2}\right)^2 + 1\right]$$
$$= 2[8 + 60 + 30 + 1]$$
$$= 2[99]$$
$$= 198$$

$$\therefore \quad (x+1)^6 + (x-1)^6 = 198$$

Binomial Theorem Ex 18.1 Q5(i)

We have,

$$(96)^3 = (100-4)^3$$
$$= {}^3C_0 \times 100^3 + {}^3C_1 \times 100^2 \times (-4) + {}^3C_2 \times 100 \times (-4)^2 + {}^3C_3 \times (-4)^3$$
$$= 100^3 - 3 \times 100^2 \times 4 + 3 \times 100 \times 4^2 - 4^3$$
$$= 1000000 - 120000 + 4800 - 64$$
$$= 1004800 - 120064$$
$$= 884736$$

$$\therefore \quad (96)^3 = 884736$$

Binomial Theorem Ex 18.1 Q5(ii)

We have,

$$(102)^5 = (100+2)^5$$
$$= {}^5C_0 \times 100^5 + {}^5C_1 \times 100^4 \times 2 + {}^5C_2 \times 100^3 \times 2^2 + {}^5C_3 \times 100^2 \times 2^3 + {}^5C_4 \times 100 \times 2^4 + {}^5C_5 \times 2^5$$
$$= 100^5 + 5 \times 100^4 \times 2 + 10 \times 100^3 \times 2^2 + 10 \times 100^2 \times 2^3 + 5 \times 100 \times 2^4 + 2^5$$
$$= 10000000000 + 1000000000 + 40000000 + 800000 + 8000 + 32$$
$$= 11040808032$$

$$\therefore \quad (102)^5 = 11040808032$$

Binomial Theorem Ex 18.1 Q5(iii)

We have,

$$(101)^4 = (100+1)^4$$
$$= {}^4C_0 \times 100^4 + {}^4C_1 \times 100^3 + {}^4C_2 \times 100^2 + {}^4C_3 \times 100 + {}^4C_4$$
$$= 100^4 + 4 \times 100^3 + 6 \times 100^2 + 4 \times 100 + 1$$
$$= 100000000 + 4000000 + 60000 + 400 + 1$$
$$= 104060401$$

$\therefore \quad (101)^4 = 104060401$

Binomial Theorem Ex 18.1 Q5(iv)

We have,

$$(98)^5 = (100-2)^5$$
$$= {}^5C_0 \times 100^5 + {}^5C_1 \times 100^4 \times (-2) + {}^5C_2 \times 100^3 \times (-2)^2 + {}^5C_3 \times 100^2 \times (-2)^3 + {}^5C_4 \times 100 \times (-2)^4 + {}^5C_5 \times (-2)^5$$
$$= {}^5C_0 \times 100^5 - {}^5C_1 \times 100^4 \times 2 + {}^5C_2 \times 100^3 \times 4 - {}^5C_3 \times 100^2 \times 8 + {}^5C_4 \times 100 \times 16 - {}^5C_5 \times 32$$
$$= 100^5 - 10 \times 100^4 + 40 \times 100^3 - 80 \times 100^2 + 80 \times 100 - 32$$
$$= 10000000000 - 1000000000 + 40000000 - 800000 + 8000 - 32$$
$$= 10040008000 - 1000800032$$
$$= 9039207968$$

$\therefore \quad (98)^5 = 9039207968$

Binomial Theorem Ex 18.1 Q6

$$2^{3n} - 7n - 1$$
$$= 2^{3(n)} - 7(n) - 1$$
$$= 8^n - 7n - 1$$
$$= (1+7)^n - 7n - 1$$
$$= \left({}^nC_0 + {}^nC_1(7)^1 + {}^nC_2(7)^2 + \ldots\ldots {}^nC_n(7)^n\right) - 7n - 1$$
$$= \left(1 + 7n + 49\,{}^nC_2 + \ldots\ldots + 49(7)^{n-2}\right) - 7n - 1$$
$$= 49\left({}^nC_2 + \ldots\ldots + 7^{n-2}\right)$$

$\therefore$ **$2^{3n} - 7n - 1$ is divisible by 49**

Hence, proved

Binomial Theorem Ex 18.1 Q7

$$3^{2n+2} - 8n - 9$$
$$= 3^{2(n+1)} - 8n - 9$$
$$= 9^{n+1} - 8n - 9$$
$$= (1+8)^{n+1} - 8n - 9$$
$$= \left({}^{n+1}C_0 + {}^{n+1}C_1 8^1 + {}^{n+1}C_2 8^2 + \ldots\ldots + {}^{n+1}C_{n+1} 8^{n+1}\right) - 8n - 9$$
$$= \left(1 + 8(n+1) + 64\,{}^{n+1}C_2 + \ldots\ldots + 64(8)^{n-1}\right) - 8n - 9$$
$$= 64\left({}^{n+1}C_2 + \ldots\ldots + 8^{n-1}\right)$$

Thus, $3^{2n+2} - 8n - 9$ is divisible by 64.

Binomial Theorem Ex 18.1 Q8

$3^{3n}-26n-1$

$=\left(3^3\right)^n-26n-1$

$=27^n-26n-1$

$=(1+26)^n-26n-1$

$=\left({}^nC_0+{}^nC_1(26)^1+{}^nC_2(26)^2+......+{}^nC_n(26)^n\right)-26n-1$

$=\left(1+26n+676\,{}^nC_2+......+676(26)^{n-2}\right)-26n-1$

$=676\left({}^nC_2+......+(26)^{n-2}\right)$

$\therefore$ $3^{3n}-26n-1$ is divisible for $n\in N$.

Hence, proved

Binomial Theorem Ex 18.1 Q9

We have,

$(1.1)^{10000}=(1+0.1)^{10000}$

$={}^{10000}C_0+{}^{10000}C_1(0.1)+{}^{10000}C_2(0.1)^2+\ldots+{}^{10000}C_{10000}(0.1)^{10000}$

$=1+10000\times(0.1)+$ other positive terms

$=1+1000+$ other positive terms

$=1001+$ other positive terms >1000

$\therefore$ $(1.1)^{10000}>1000$

Binomial Theorem Ex 18.1 Q10

$(1.2)^{4000}=(1+0.2)^{4000}$

$={}^{4000}C_0(0.2)^0(1)^{4000}+{}^{4000}C_1\times(0.2)^1\times1^{3999}+........+{}^{4000}C_{400}(0.2)^{4000}1^0$

$=1+4000\times0.2\times1+........+(0.2)^{4000}$

$=1+800+........+(0.2)^{4000}$

Here, we clearly observe $(1,2)^{4000}$ is less **than (801) thus, $(1.2)^{4000}<800$.**

Binomial Theorem Ex 18.1 Q11

$$(1.01)^{10}+(1-0.01)^{10}=(1+0.01)^{10}+(1-0.01)^{10}$$

$$=\left({}^{10}C_1+{}^{10}C_2\frac{1}{10^2}+{}^{10}C_3\frac{1}{10^3}\ldots+{}^{10}C_{10}\frac{1}{10^{10}}\right)+\left({}^{10}C_1-{}^{10}C_2\frac{1}{10^2}+{}^{10}C_3\frac{1}{10^3}-{}^{10}C_4\frac{1}{10^4}+\ldots\right)$$

$$=2\left({}^{10}C_1-{}^{10}C_3\frac{1}{10^3}+{}^{10}C_5\frac{1}{10^5}+{}^{10}C_7\frac{1}{10^7}+{}^{10}C_9\frac{1}{10^9}\right)$$

$$=2\left(10+\frac{10!}{3!7!}\frac{1}{1000}+\frac{10!}{5!5!}\frac{1}{(10)^5}+\frac{10!}{7!3!}\times\frac{1}{10^7}+\frac{10!}{9!1!}\frac{1}{10^9}\right)$$

$$=2\left(10+\frac{9\times8}{3\times2\times1000}+\frac{9\times8\times7\times6}{5\times4\times3\times2\times10^5}+\frac{9\times8}{3\times2\times10^7}+\frac{1}{10^8}\right)$$

$$=2.0090042$$

Binomial Theorem Ex 18.1 Q12

$$2^{4n+4}-15n-16=2^{4(n+1)}-15n-15-1$$

$$=(16)^{(n+1)}-15(n+1)-1$$

$$=(1+15)^{n+1}-15(n+1)-1$$

$$=\left[{}^{n+1}C_0+{}^{n+1}C_1(15)+{}^{n+1}C_2(15)^2+\ldots\ldots+{}^{n+1}C_{n+1}(15)^{n+1}\right]-15(n+1)-1$$

$$=\left[1+15(n+1)+{}^{n+1}C_2(15)^2+\ldots\ldots+{}^{n+1}C_{n+1}(15)^{n+1}\right]-15(n+1)-1$$

$$=225\left[{}^{n+1}C_2+\ldots\ldots+{}^{n+1}C_{n+1}(15)^{n-1}\right]$$

$$=225\times\text{natural number}$$