RD Sharma Solutions Class 11 Maths Chapter 13 Ex 13.1

Complex numbers Ex 13.1 Q1(i)

We know that $i = \sqrt{-1}$

$$i^2 = -1$$
$$i^3 = -i$$
$$i^4 = 1$$

In order to find i^n where $n > 4$, we divide n by 4 to get quotient p and remainder q, so that

$$n = 4p + q, o \le q < 4$$

Then $i^n = i^{4p+q}$

$$= i^{4p} \times i^q$$
$$= \left(i^4\right)^p \times i^q$$
$$= 1^p \times i^q$$
$$= i^q \qquad \left[\because 1^{p=1}\right]$$

Hence $i^n = i^q$, *where* $o \le q < 4$

$$\therefore i^{457} = i^{4\times 114} \times i^1$$
$$= i^1$$
$$= i$$

Complex numbers Ex 13.1 Q1(ii)

We know that $i = \sqrt{-1}$

$$i^2 = -1$$
$$i^3 = -i$$
$$i^4 = 1$$

In order to find i^n where $n > 4$, we divide n by 4 to get quotient p and remainder q, so that

$$n = 4p + q, o \le q < 4$$

Then $i^n = i^{4p+q}$

$$= i^{4p} \times i^q$$
$$= \left(i^4\right)^p \times i^q$$
$$= 1^p \times i^q$$
$$= i^q \qquad \left[\because 1^{p=1}\right]$$

Hence $i^n = i^q$, *where* $o \le q < 4$

$$\therefore i^{528} = i^{4\times 132}$$
$$= \left(i^4\right)^{132}$$
$$= 1^{132}$$
$$= 1$$

$$\therefore \left(i^{528}\right) = 1$$

Complex numbers Ex 13.1 Q1(iii)

We know that $i = \sqrt{-1}$

$$i^2 = -1$$
$$i^3 = -i$$
$$i^4 = 1$$

In order to find i^n where $n > 4$, we divide n by 4 to get quotient p and remainder q, so that

$$n = 4p + q, o \le q < 4$$

Then $i^n = i^{4p+q}$

$$= i^{4p} \times i^q$$
$$= \left(i^4\right)^p \times i^q$$
$$= 1^p \times i^q$$
$$= i^q \qquad \left[\because 1^{p-1}\right]$$

Hence $i^n = i^q$, *where* $o \le q < 4$

$$\therefore \frac{1}{i^{58}} = \frac{1}{i^{4\times 14} \times i^2}$$
$$= \frac{1}{1 \times i^2}$$
$$= \frac{1}{-1} \qquad \left[\because i^2 = -1\right]$$
$$= -1$$

Complex numbers Ex 13.1 Q1(iv)

We know that $i = \sqrt{-1}$

$$i^2 = -1$$
$$i^3 = -i$$
$$i^4 = 1$$

In order to find i^n where $n > 4$, we divide n by 4 to get quotient p and remainder q, so that

$$n = 4p + q, o \le q < 4$$

Then $i^n = i^{4p+q}$

$$= i^{4p} \times i^q$$
$$= \left(i^4\right)^p \times i^q$$
$$= 1^p \times i^q$$
$$= i^q \qquad \left[\because 1^{p=1}\right]$$

Hence $i^n = i^q$, *where* $o \le q < 4$

$$\therefore\ i^{37} + \frac{1}{i^{67}} = i^{4\times 9} \times i^1 + \frac{1}{i^{4\times 16} \times i^3}$$
$$= 1 \times i^1 + \frac{1}{1 \times i^3}$$
$$= i + \frac{1}{i^3 \times i} \times i$$
$$= i + \frac{i}{i^4}$$
$$= i + \frac{i}{1} \qquad \left[\because i^4 = 1\right]$$
$$= 2i$$

$$\therefore\ i^{37} + \frac{1}{i^{67}} = 2i$$

Complex numbers Ex 13.1 Q1(v)

We know that $i = \sqrt{-1}$

$$i^2 = -1$$
$$i^3 = -i$$
$$i^4 = 1$$

In order to find i^n where $n > 4$, we divide n by 4 to get quotient p and remainder q, so that

$$n = 4p + q, o \le q < 4$$

Then $i^n = i^{4p+q}$

$$= i^{4p} \times i^q$$
$$= \left(i^4\right)^p \times i^q$$
$$= 1^p \times i^q$$
$$= i^q \qquad \left[\because 1^{p=1}\right]$$

Hence $i^n = i^q$, *where* $o \le q < 4$

$$\left(i^{41} + \frac{1}{i^{257}}\right)^9 = \left(i^{4\times10} \times i^1 + \frac{1}{i^{4\times64} \times i^1}\right)^9$$
$$= \left(1 \times i + \frac{1}{1 \times i}\right)^9$$
$$= \left(i + \frac{1}{i}\right)^9$$
$$= \left(i + \frac{1}{i \times i} \times i\right)^9$$
$$= \left(i + \frac{i}{-1}\right)^9$$
$$= (i - i)^9$$
$$= 0$$

Complex numbers Ex 13.1 Q1(vi)

We know that $i = \sqrt{-1}$

$$i^2 = -1$$
$$i^3 = -i$$
$$i^4 = 1$$

In order to find i^n where $n > 4$, we divide n by 4 to get quotient p and remainder q, so that

$$n = 4p + q, o \le q < 4$$

Then $i^n = i^{4p+q}$

$$= i^{4p} \times i^q$$
$$= \left(i^4\right)^p \times i^q$$
$$= 1^p \times i^q$$
$$= i^q \qquad \left[\because 1^{p-1}\right]$$

Hence $i^n = i^q$, *where* $o \le q < 4$

$$\left(i^{77} + i^{70} + i^{87} + i^{414}\right)^3 = \left(i^{4\times19} \times i^1 + i^{4\times17} \times i^2 + i^{4\times21} \times i^3 + i^{4\times103} \times i^2\right)^3$$
$$= \left(1 \times i + 1 \times i^2 + 1 \times i^3 + 1 \times i^2\right)^3$$
$$= \left(i - 1 - i - 1\right)^3$$
$$= \left(-2\right)^3$$
$$= -8$$

$$\therefore \left(i^{77} + i^{70} + i^{87} + i^{414}\right)^3 = -8$$

Complex numbers Ex 13.1 Q1(vii)

We know that $i = \sqrt{-1}$

$$i^2 = -1$$
$$i^3 = -i$$
$$i^4 = 1$$

In order to find i^n where $n > 4$, we divide n by 4 to get quotient p and remainder q, so that

$$n = 4p + q, o \le q < 4$$

Then $i^n = i^{4p+q}$

$$= i^{4p} \times i^q$$
$$= \left(i^4\right)^p \times i^q$$
$$= 1^p \times i^q$$
$$= i^q \qquad \left[\because 1^{p-1}\right]$$

Hence $i^n = i^q$, *where* $o \le q < 4$

$$\therefore i^{30} + i^{40} + i^{60} = i^{4\times7} \times i^2 + i^{4\times10} + i^{4\times15}$$
$$= 1 \times i^2 + 1 + 1$$
$$= -1 + 1 + 1$$
$$= 1$$

$$\therefore i^{30} + i^{40} + i^{60} = 1$$

Complex numbers Ex 13.1 Q1(viii)

We know that $i = \sqrt{-1}$

$i^2 = -1$

$i^3 = -i$

$i^4 = 1$

In order to find i^n where $n > 4$, we divide n by 4 to get quotient p and remainder q, so that

$$n = 4p + q, o \le q < 4$$

Then $i^n = i^{4p+q}$

$= i^{4p} \times i^q$

$= \left(i^4\right)^p \times i^q$

$= 1^p \times i^q$

$= i^q \qquad \left[\because 1^{p=1}\right]$

Hence $i^n = i^q$, *where* $o \le q < 4$

$i^{49} + i^{68} + i^{89} + i^{110} = i^{4\times12} \times i^1 + i^{4\times17} + i^{4\times22} \times i^1 + i^{4\times27} \times i^2$

$= 1 \times i + 1 + 1 \times i + 1 \times i^2$

$= i + 1 + i - 1$

$= 2i$

$\therefore i^{49} + i^{68} + i^{89} + i^{110} = 2i$

Complex numbers Ex 13.1 Q2

$1 + i^{10} + i^{20} + i^{30} = 1 + i^{4\times2} \times i^2 + i^{4\times5} + i^{4\times7} \times i^2$

$= 1 + 1 \times i^2 + 1 + 1 \times i^2$

$= 1 - 1 + 1 - 1$

$= 0$, which is real number

Complex numbers Ex 13.1 Q3(i)

$i^{49} + i^{68} + i^{89} + i^{110} = i^{4\times12} \times + i^1 + i^{4\times17} + i^{4\times22} \times i^1 + i^{4\times27} \times i^2$

$= 1 \times i + 1 + 1 \times i + 1 \times i^2$

$= i + 1 + i - 1$

$= 2i$

$\therefore i^{49} + i^{68} + i^{89} + i^{110} = 2i$

Complex numbers Ex 13.1 Q3(ii)

$i^{30} + i^{80} + i^{120} = i^{4\times7} \times i^2 + i^{4\times20} + i^{4\times30}$

$= 1 \times i^2 + 1 + 1$

$= -1 + 1 + 1$

$= 1$

$\therefore i^{30} + i^{80} + i^{120} = 1$

Complex numbers Ex 13.1 Q3(iii)

$$i + i^2 + i^3 + i^4 = 1 + (-1) + (-i) + 1$$
$$= 0$$

$$\therefore\ i + i^2 + i^3 + i^4 = 0$$

Complex numbers Ex 13.1 Q3(iv)

$$i^5 + i^{10} + i^{15} = i^{4\times 1} \times i^1 + i^{4\times 2} \times i^2 + i^{4\times 3} \times i^3$$
$$= 1 \times i + 1 \times i^2 + 1 \times i^3$$
$$= i - 1 - i$$
$$= -1$$

$$\therefore\ i^5 + i^{10} + i^{15} = -1$$

Complex numbers Ex 13.1 Q3(v)

$$\frac{i^{592} + i^{590} + i^{588} + i^{586} + i^{584}}{i^{582} + i^{580} + i^{578} + i^{576} + i^{574}} = \frac{i^{4\times 148} + i^{147} \times i^2 + i^{4\times 147} + i^{4\times 146} \times i^2 + i^{4\times 146}}{i^{4\times 145} \times i^2 + i^{4\times 145} + i^{4\times 144} \times i^2 + i^{4\times 144} + i^{4\times 143} \times i^2}$$
$$= \frac{1 + 1 \times i^2 + 1 + 1 \times i^2 + 1}{1 \times i^2 + 1 + 1 \times i^2 + 1 + 1 \times i^2}$$
$$= \frac{1 - 1 + 1 - 1 + 1}{-1 + 1 - 1 + 1 - 1}$$
$$= \frac{1}{-1}$$
$$= -1$$

$$\therefore\ \frac{i^{592} + i^{590} + i^{588} + i^{586} + i^{584}}{i^{582} + i^{580} + i^{578} + i^{576} + i^{574}} = -1$$

Complex numbers Ex 13.1 Q3(vi)

$$1 + i^2 + i^4 + i^6 + i^8 + \ldots + i^{20}$$
$$= 1 + i^2 + i^4 + i^{4\times 1} \times i^2 + i^{4\times 2} + i^{4\times 2} \times i^2 + i^{4\times 3} + i^{4\times 3} \times i^2 + i^{4\times 4} + i^{4\times 4} \times i^2 + i^{4\times 5}$$
$$= 1 - 1 + 1 + 1 \times i^2 + 1 + 1 \times i^2 + 1 + 1 \times i^2 + 1 + 1 \times i^2 + 1$$
$$= 1 - 1 + 1 - 1 + 1 - 1 + 1 - 1 + 1 - 1 + 1$$
$$= 1$$

Complex numbers Ex 13.1 Q3(vii)

$$(1 + i)^6 + (1 - i)^3 = \left[(1 + i)^2\right]^3 + (1 - i)^3$$
$$= (1 + i^2 + 2i)^3 + (1 - 3i + 3i^2 - i^3)$$
$$= (1 - 1 + 2i)^3 + (1 - 3i - 3 + i)$$
$$= 8i^3 - 2 - 2i$$
$$= -8i - 2 - 2i$$
$$= -2 - 10i$$