

RD SHARMA

Solutions

Class 9 Maths

Chapter 10

Ex 10.2

(1) In fig. (10).40, it is given that $RT = TS$, $\angle 1 = 2 \angle 2$ and $4 = 2 \angle 3$ Prove that $\triangle RBT \cong \triangle SAT$.

Solution:

In the figure, given that

$$RT = TS \quad \dots(i)$$

$$\angle 1 = 2 \angle 2 \quad \dots(ii)$$

$$\text{And } \angle 4 = 2 \angle 3 \quad \dots(iii)$$

To prove that $\triangle RBT \cong \triangle SAT$.

Let the point of intersection RB and SA be denoted by O

Since RB and SA intersect at O

$$\angle AOR = \angle BOS \quad [\text{Vertically opposite angles}]$$

$$\angle 1 = \angle 4$$

$$2 \angle 2 = 2 \angle 3 \quad [\text{From (ii) and (iii)}]$$

$$\angle 2 = \angle 3 \quad \dots(iv)$$

Now we have $RT = TS$ in $\triangle TRS$

$\triangle TRS$ is an isosceles triangle

$$\angle TRS = \angle TSR \quad \dots(v)$$

But we have

$$\angle TRS = \angle TRB + \angle 2 \quad \dots(vi)$$

$$\angle TSR = \angle TSA + \angle 3 \quad \dots(vii)$$

Putting (vi) and (vii) in (v) we get

$$\angle TRB + \angle 2 = \angle TSA + \angle 3$$

$$\Rightarrow \angle TRB = \angle TSA \quad [\text{From (iv)}]$$

Now consider $\triangle RBT$ and $\triangle SAT$

$$RT = ST \quad [\text{From (i)}]$$

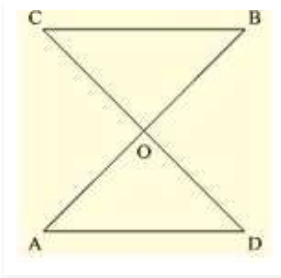
$$\angle TRB = \angle TSA \quad [\text{From (iv)}] \quad \angle RTB = \angle STA \quad [\text{Common angle}]$$

From ASA criterion of congruence, we have

$$\triangle RBT = \triangle SAT$$

(2) Two lines AB and CD intersect at O such that BC is equal and parallel to AD. Prove that the lines AB and CD bisect at O.

Solution: Given that lines AB and CD Intersect at O



Such that $BC \parallel AD$ and $BC = AD$ (i)

We have to prove that AB and CD bisect at O.

To prove this first we have to prove that $\triangle AOD \cong \triangle BOC$

(3) BD and CE are bisectors of $\angle B$ and $\angle C$ of an isosceles $\triangle ABC$ with $AB = AC$. Prove that $BD = CE$

Solution:

Given that $\triangle ABC$ is isosceles with $AB = AC$ and BD and CE are bisectors of $\angle B$ and $\angle C$ We have to prove $BD = CE$

Since $AB = AC$

$\Rightarrow \angle ABC = \angle ACB$ (i)

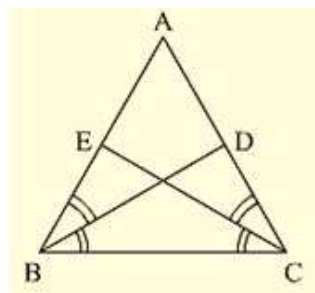
[Angles opposite to equal sides are equal]

Since BD and CE are bisectors of $\angle B$ and $\angle C$

$$\angle ABD = \angle DBC = \angle BCE = \angle ECA = \frac{\angle B}{2} = \frac{\angle C}{2}$$

Now,

Consider $\triangle EBC = \triangle DCB$



$\angle EBC = \angle DCB$ [$\angle B = \angle C$] [From (i)]

$BC = BC$ [Common side]

$\angle BCE = \angle CBD$ [From (ii)]

So, by ASA congruence criterion, we have $\triangle EBC \cong \triangle DCB$

Now,

$CE = BD$ [Corresponding parts of congruent triangles are equal]

or, $BD = CE$

Hence proved

Since $AD \parallel BC$ and transversal AB cuts at A and B respectively

$$\angle DAO = \angle OBC \text{(ii) [alternate angle]}$$

And similarly $AD \parallel BC$ and transversal DC cuts at D and C respectively

$$\angle ADO = \angle OCB \text{(iii) [alternate angle]}$$

Since AB and CD intersect at O .

$$\angle AOD = \angle BOC \text{ [Vertically opposite angles]}$$

Now consider $\triangle AOD$ and $\triangle BOC$

$$\angle DAO = \angle OCB \text{ [From (ii)]}$$

$$AD = BC \text{ [From (i)]}$$

$$\text{And } \angle ADO = \angle OCB \text{ [From (iii)]}$$

So, by ASA congruence criterion, we have

$$\triangle AOD \cong \triangle BOC$$

Now,

$$AO = OB \text{ and } DO = OC \text{ [Corresponding parts of congruent triangles are equal]}$$

Lines AB and CD bisect at O .

Hence proved