RD SHARMA

Solutions

Class 9 Maths

Chapter 1

Ex *1.4*

Q1. Define an irrational number.

Solution:

An irrational number is a real number which can be written as a decimal but not as a fraction i.e. it cannot be expressed as a ratio of integers. It cannot be expressed as terminating or repeating decimal.

Q2. Explain how an irrational number is differing from rational numbers?

Solution: An irrational number is a real number which can be written as a decimal but not as a fraction i.e. it cannot be expressed as a ratio of integers. It cannot be expressed as terminating or repeating decimal.

For example, 0.10110100 is an irrational number

A rational number is a real number which can be written as a fraction and as a decimal i.e. it can be expressed as a ratio of integers. . It can be expressed as terminating or repeating decimal.

For examples,

0.10 and $0.\overline{4}$ both are rational numbers

Q3. Find, whether the following numbers are rational and irrational

(i) $\sqrt{7}$

(ii) $\sqrt{4}$

(iii) $2 + \sqrt{3}$

(iv) $\sqrt{3} + \sqrt{2}$

(v) $\sqrt{3} + \sqrt{5}$

(vi) $(\sqrt{2} – 2)^2$

(vii) $(2 – \sqrt{2})(2 + \sqrt{2})$

(viii) $(\sqrt{2} + \sqrt{3})^2$

(ix) $\sqrt{5}$ – 2

(x) $\sqrt{23}$

(xi) $\sqrt{225}$

(xii) 0.3796

(xiii) 7.478478……

(xiv) 1.101001000100001……

Solution:

(i) $\sqrt{7}$ is not a perfect square root so it is an Irrational

number.

(ii) $\sqrt{4}$ is a perfect square root so it is an rational number.

We have,

$\sqrt{4}$ can be expressed in the form of

$\frac{a}{b}$, so it is a rational number. The decimal

representation of $\sqrt{9}$ is 3.0. 3 is a rational number.

(iii) $2 + \sqrt{3}$

Here, 2 is a rational number and $\sqrt{3}$ is an irrational number

So, the sum of a rational and an irrational number is an irrational number.

(iv) $\sqrt{3} + \sqrt{2}$

$\sqrt{3}$ is not a perfect square and it is an irrational number and $\sqrt{2}$ is not a perfect square and is an irrational number. The sum of an irrational number and an irrational number is an irrational number, so $\sqrt{3} + \sqrt{2}$ is an irrational number.

(v) $\sqrt{3} + \sqrt{5}$

$\sqrt{3}$ is not a perfect square and it is an irrational number and $\sqrt{5}$ is not a perfect square and is an irrational number. The sum of an irrational number and an irrational number is an irrational number, so $\sqrt{3} + \sqrt{5}$ is an irrational number.

(vi) $(\sqrt{2} – 2)^2$

We have, $(\sqrt{2} – 2)^2$

$= 2 + 4 - 4\sqrt{2}$

$= 6 + 4\sqrt{2}$

6 is a rational number but $4\sqrt{2}$ is an irrational number.

The sum of a rational number and an irrational number is an irrational number, so $(\sqrt{2} + \sqrt{4})^2$ is an irrational number.

(vii) $(2 – \sqrt{2})(2 + \sqrt{2})$

We have,

$(2 – \sqrt{2})(2 + \sqrt{2}) = (2)^2 - (\sqrt{2})^2$ [Since, $(a + b)(a - b) = a^2 - b^2$]

$4 - 2 = \frac{2}{1}$

Since, 2 is a rational number.

$(2 – \sqrt{2})(2 + \sqrt{2})$ is a rational number.

(viii) $(\sqrt{2} + \sqrt{3})^2$

We have,

$(\sqrt{2} + \sqrt{3})^2 = 2 + 2\sqrt{6} + 3 = 5 + \sqrt{6}$ [Since, $(a+b)^2 = a^2 + 2ab + b^2$

The sum of a rational number and an irrational number is an irrational number, so $(\sqrt{2}+\sqrt{3})^2$ is an irrational number.

(ix) $\sqrt{5}$ – 2

The difference of an irrational number and a rational number is an irrational number.

($\sqrt{5}$ – 2) is an irrational number.

(x) $\sqrt{23}$

$\sqrt{23}$ = 4.795831352331…

As decimal expansion of this number is non-terminating, non-recurring so it is an irrational number.

(xi) $\sqrt{225}$

$\sqrt{225}$ = 15 = $\frac{15}{1}$

$\sqrt{225}$ is rational number as it can be represented in $\frac{p}{q}$ form.

(xii) 0.3796

0.3796, as decimal expansion of this number is terminating, so it is a rational number.

(xiii) 7.478478……

7.478478 = 7.478, as decimal expansion of this number is non-terminating recurring so it is a rational number.

(xiv) 1.101001000100001……

1.101001000100001……, as decimal expansion of this number is non-terminating, non-recurring so it is an irrational number

Q4. Identify the following as irrational numbers. Give the decimal representation of rational numbers:

(i) $\sqrt{4}$

(ii) 3 $\times\sqrt{18}$

(iii) sqrt1.44

(iv) $\sqrt{\frac{9}{27}}$

(v) – $\sqrt{64}$

(vi) $\sqrt{100}$

Solution:

(i) We have,

$\sqrt{4}$ can be written in the form of

$\frac{p}{q}$. So, it is a rational number. Its decimal

representation is 2.0

(ii). We have,

$3 \times \sqrt{18}$

$= 3 \times \sqrt{2x3x3}$

$= 9 \times \sqrt{2}$

Since, the product of a ratios and an irrational is an irrational number.

$9 \times \sqrt{2}$is an irrational.

$3 \times \sqrt{18}$ is an irrational number.

(iii) We have,

sqrt1.44

$= \sqrt{\frac{144}{100}}$

$= \frac{12}{10}$

= 1.2

Every terminating decimal is a rational number, so 1.2 is a rational number.

Its decimal representation is 1.2.

(iv) $\sqrt{\frac{9}{27}}$

We have,

$\sqrt{\frac{9}{27}}$

$= \frac{3}{\sqrt{27}}$

$= \frac{1}{\sqrt{3}}$

Quotient of a rational and an irrational number is irrational numbers so

$\frac{1}{\sqrt{3}}$ is an irrational number.

$\sqrt{\frac{9}{27}}$ is an irrational number.

(v) We have,

$-\sqrt{64}$

$= -8$

$= -\frac{8}{1}$

$= -\frac{8}{1}$ can be expressed in the form of $\frac{a}{b}$,

so $-\sqrt{64}$ is a rational number.

Its decimal representation is − 8.0.

(vi) We have,

$\sqrt{100}$

= 10 can be expressed in the form of $\frac{a}{b}$,

so $\sqrt{100}$ is a rational number

Its decimal representation is 10.0.

Q5. In the following equations, find which variables x, y and z etc. represent rational or irrational numbers:

(i) x^2 = 5

(ii) y^2 = 9

(iii) z^2 = 0.04

(iv) u^2 = $\frac{17}{4}$

(v) v^2 = 3

(vi) w^2 = 27

(vii) t^2 = 0.4

Solution:

(i) We have,

x^2 = 5

Taking square root on both the sides, we get

x = $\sqrt{5}$

$\sqrt{5}$ is not a perfect square root, so it is an irrational number.

(ii) We have,

=y^2 = 9

= 3

= $\frac{3}{1}$ can be expressed in the form of $\frac{a}{b}$, so it a rational number.

(iii) We have,

z^2 = 0.04

Taking square root on the both sides, we get

z = 0.2

$\frac{2}{10}$ can be expressed in the form of $\frac{a}{b}$, so it is a rational number.

(iv) We have,

u^2 = $\frac{17}{4}$

Taking square root on both sides, we get,

$u = \sqrt{\frac{17}{4}}$

$u = \frac{\sqrt{17}}{2}$

Quotient of an irrational and a rational number is irrational, so u is an Irrational number.

(v) We have,

$v^2 = 3$

Taking square root on both sides, we get,

$v = \sqrt{3}$

$\sqrt{3}$ is not a perfect square root, so v is irrational number.

(vi) We have,

$w^2 = 27$

Taking square root on both the sides, we get,

$w = 3\sqrt{3}$

Product of a irrational and an irrational is an irrational number. So w is an irrational number.

(vii) We have,

$t^2 = 0.4$

Taking square root on both sides, we get,

$t = \sqrt{\frac{4}{10}}$

$t = \frac{2}{\sqrt{10}}$

Since, quotient of a rational and an Irrational number is irrational

number. $t^2 = 0.4$ is an irrational number.

Q6. Give an example of each, of two irrational numbers whose:

(i) Difference in a rational number.

(ii) Difference in an irrational number.

(iii) Sum in a rational number.

(iv) Sum is an irrational number.

(v) Product in a rational number.

(vi) Product in an irrational number.

(vii) Quotient in a rational number.

(viii) Quotient in an irrational number.

Solution:

(i) $\sqrt{2}$ is an irrational number.

Now, $\sqrt{2} - \sqrt{2}$ = 0.

0 is the rational number.

(ii) Let two irrational numbers are $3\sqrt{2}$ and $\sqrt{2}$.

$3\sqrt{2} - \sqrt{2} = 2\sqrt{2}$

$5\sqrt{6}$ is the rational number.

(iii) $\sqrt{11}$ is an irrational number.

Now, $\sqrt{11}$ + (- $\sqrt{11}$) = 0.

0 is the rational number.

(iv) Let two irrational numbers are $4\sqrt{6}$ and $\sqrt{6}$

$4\sqrt{6} + \sqrt{6}$

$5\sqrt{6}$ is the rational number.

(iv) Let two Irrational numbers are $7\sqrt{5}$ and

$\sqrt{5}$

Now, $7\sqrt{5} \times \sqrt{5}$

= 7 × 5

= 35 is the rational number.

(v) Let two irrational numbers are $\sqrt{8}$ and $\sqrt{8}$.

Now, $\sqrt{8} \times \sqrt{8}$

8 is the rational number.

(vi) Let two irrational numbers are $4\sqrt{6}$ and $\sqrt{6}$

Now, $\frac{4\sqrt{6}}{\sqrt{6}}$

= 4 is the rational number

(vii) Let two irrational numbers are $3\sqrt{7}$ and $\sqrt{7}$

Now, 3 is the rational number.

(viii) Let two irrational numbers are $\sqrt{8}$ and $\sqrt{2}$

Now $\sqrt{2}$ is an rational number.

Q7. Give two rational numbers lying between 0.232332333233332 and 0.212112111211112.

Solution: Let a = 0.212112111211112

And, b = 0.232332333233332…

Clearly, a < b because in the second decimal place a has digit 1 and b has digit 3 If we consider rational numbers in which the second decimal place has the digit 2, then they will lie between a and b.

Let. x = 0.22

y = 0.22112211… Then a < x < y < b

Hence, x, and y are required rational numbers.

Q8. Give two rational numbers lying between 0.515115111511115 and 0. 5353353335

Solution: Let, a = 0.515115111511115…

And, b = 0.5353353335..

We observe that in the second decimal place a has digit 1 and b has digit 3, therefore, a < b.

So If we consider rational numbers

x = 0.52

y = 0.52062062…

We find that,

a < x < y < b

Hence x and y are required rational numbers.

Q9. Find one irrational number between 0.2101 and 0.2222 … = $0.\overline{2}$

Solution:

Let, a = 0.2101 and,

b =0.2222…

We observe that in the second decimal place a has digit 1 and b has digit 2, therefore a < b in the third decimal place a has digit 0.

So, if we consider irrational numbers

x = 0.211011001100011….

We find that a < x < b

Hence x is required irrational number.

Q10. Find a rational number and also an irrational number lying between the numbers 0.3030030003… and 0.3010010001…

Solution: Let,

a=0.3010010001 and,

b = 0.3030030003…

We observe that in the third decimal place a has digit 1 and b has digit

3, therefore a < b in the third decimal place a has digit 1. So, if we

consider rational and irrational numbers x=0.302

y = 0.302002000200002…..

We find that a < x < b and, a < y < b.

Hence, x and y are required rational and irrational numbers respectively.

Q11. Find two irrational numbers between 0.5 and 0.55.

Solution: Let a = 0.5 = 0.50 and b =0.55

We observe that in the second decimal place a has digit 0 and b has digit

5, therefore a < 0 so, if we consider irrational numbers

x =0.51051005100051…

y = 0.530535305353530…

We find that a < x < y < b

Hence x and y are required irrational numbers.

Q12. Find two irrational numbers lying between 0.1 and 0.12.

Solution:

Let a = 0.1 =0.10

And b = 0.12

We observe that In the second decimal place a has digit 0 and b has digit 2.

Therefore, a < b.

So, if we consider irrational numbers

x = 0.1101101100011… y=0.111011110111110… We find that a < x < y < 0

Hence, x and y are required irrational numbers.

Q13. Prove that $\sqrt{3}+\sqrt{5}$ is an irrational number.

If possible, let $\sqrt{3}+\sqrt{5}$ be a rational number equal to x.

Then,

x= $\sqrt{3}+\sqrt{5}$

$x^2 = (\sqrt{3}+\sqrt{5})^2$

x^2 = 8 + $2\sqrt{15}$

$\frac{x^2-8}{2}$ = $\sqrt{15}$

Now, $\sqrt{\frac{x^2-8}{2}}$ is rational

$\sqrt{15}$ is rational

Thus, we arrive at a contradiction.

Hence, $\sqrt{3}+\sqrt{5}$ is an irrational number.