RD SHARMA

Solutions

Class 10 Maths

Chapter 14

Ex *14.4*

1. Find the centroid of the triangle whose vertices are:

   (i) $(1,4),(-1,-1)$ *and* $(3,-2)$

   Sol:

   We know that the coordinates of the centroid of a triangle whose vertices are $(x_1,y_1),(x_2,y_2),(x_3,y_3)$ are

   $$\left(\frac{x_1+x_2+x_3}{3},\frac{y_1+y_2+y_3}{3}\right)$$

   So, the coordinates of the centroid of a triangle whose vertices are

   $(1,4),(-1,-1)$ *and* $(3,-2)$ are $\left(\frac{1-1+3}{3},\frac{4-1-2}{3}\right)$

   $$=\left(1,\frac{1}{3}\right)$$

2. Two vertices of a triangle are (1, 2), (3, 5) and its centroid is at the origin. Find the coordinates of the third vertex.

   Sol:

Let the coordinates of the third vertex bee (x, y), Then

Coordinates of centroid of triangle are

$$\left(\frac{x+1+3}{3}, \frac{y+2+5}{3}\right)$$

We have centroid is at origin $(0,0)$

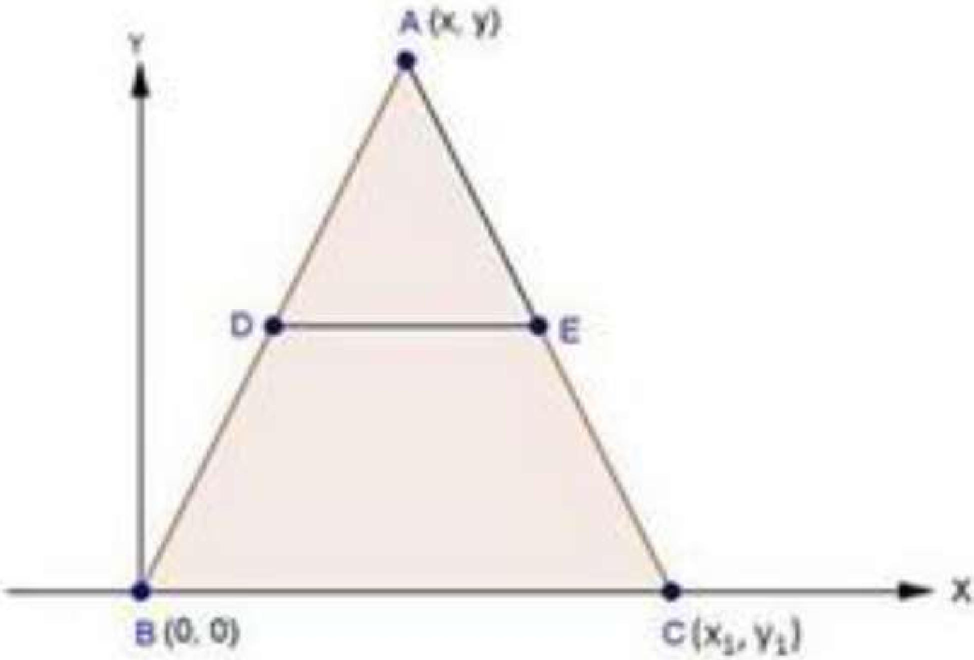
$$\therefore \frac{x+1+3}{3}=0 \text{ and } \frac{y+2+5}{3}=0$$

$\Rightarrow x+4=0 \quad \Rightarrow y+7=0$

$\Rightarrow x=-4 \quad \Rightarrow y=-7$

3. Prove analytically that the line segment joining the middle points of two sides of a triangle is equal to half of the third side.

Sol:



Let ABC be a triangle such that BC is along x-axis

Coordinates of A, B and C are $(x, y), (0,0)$ and (x_1, y_1)

D and E are the mid-points of AB and AC respectively

Coordinates of D are $\left(\frac{x+0}{2}, \frac{y+0}{2}\right)$

$$= \left(\frac{x}{2}, \frac{y}{2}\right)$$

Coordinates of E are $\left(\frac{x+x_1}{2}, \frac{y+y_1}{2}\right)$

Length of $BC = \sqrt{x_1^2 + y_1^2}$

$$\text{Length of DE } = \sqrt{\left(\frac{x+x_1}{2} - \frac{x}{2}\right)^2 + \left(\frac{x+y_1}{2} - \frac{y}{2}\right)^2}$$

$$= \sqrt{\left(\frac{x_1}{2}\right)^2 + \left(\frac{y_1}{2}\right)^2}$$

$$= \sqrt{\frac{x_1^2}{4} + \frac{y_1^2}{4}}$$

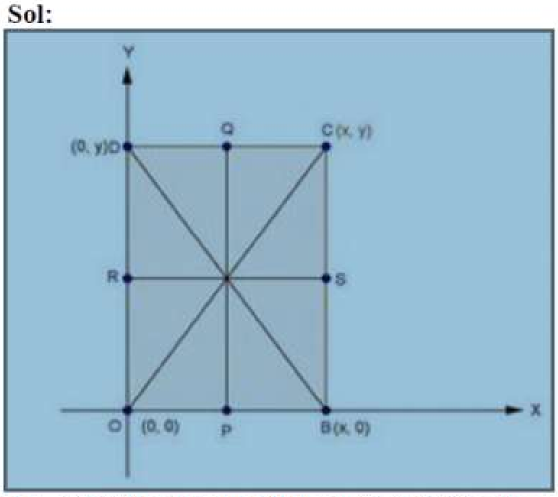
$$= \sqrt{\frac{1}{4}\left(x_1^2 + y_1^2\right)}$$

$$= \frac{1}{2}\sqrt{x_1^2 + y_1^2}$$

$$= \frac{1}{2}BC$$

4. Prove that the lines joining the middle points of the opposite sides of a quadrilateral and the join of the middle points of its diagonals meet in a point and bisect one another.

Sol:



Let $OBCD$ be the quadrilateral P, Q, R, S be the midpoint off OB, CD, OD and BC.

Let the coordinates of O, B, C, D are $(0,0), (x,0), (x,y)$ and $(0,y)$

Coordinates of P are $\left(\frac{x}{2}, 0\right)$

Coordinates of Q are $\left(\frac{x}{2}, y\right)$

Coordinates of R are $\left(0, \frac{y}{2}\right)$

Coordinates of S are $\left(x, \frac{y}{2}\right)$

Coordinates of midpoint of PQ are

$$\left[\frac{\frac{x}{2}+\frac{x}{2}}{2}, \frac{0+y}{2}\right]=\left(\frac{x}{2}, \frac{y}{2}\right)$$

Coordinates of midpoint of RS are $\left[\frac{(0+x)}{2}, \frac{\frac{y}{2}+\frac{y}{2}}{2}\right]=\left[\frac{x}{2}, \frac{y}{2}\right]$

Since, the coordinates of the mid-point of $PQ =$ coordinates of mid-point of RS

$\therefore PQ$ and RS bisect each other

5. If G be the centroid of a triangle ABC and P be any other point in the plane, prove that $PA^2 + PB^2 + PC^2 = GA^2 + GB^2 + GC^2 + 3GP^2$.

Sol:

Let $A(0,0), B(a,0)$, and $C(c,d)$ are the o-ordinates of triangle ABC

Hence, $G\left[\frac{c+0+a}{3}, \frac{d}{3}\right]$

i.e., $G\left[\frac{a+c}{3}, \frac{d}{3}\right]$

let $P(x,y)$

To prove:

$PA^2 + PB^2 + PC^2 = GA^2 + GB^2 + GC^2 + 3GP^2$

Or, $PA^2 + PB^2 + PC^2 = GA^2 + GB^2 + GC^2 + GP^2 + GP^2 + GP^2$

Or, $PA^2 - GP^2 + PB^2 - GP^2 + PC^2 + GP^2 = GA^2 + GB^2 + GC^2$

Proof:

$PA^2 = x^2 + y^2$

$GP^2 = \left(x - \frac{a+c}{3}\right)^2 + \left(y - \frac{d}{3}\right)^2$

$PB^2 = (x-a)^2 + y^2$

$PC^2 = (x-c)^2 + (y-d)^2$

L.H.S

$$= x^2 + y^2 - \left[x^2 + \left(\frac{a+c}{3}\right)^2 + 2x\frac{(a+c)}{3} + y^2 + \frac{d^2}{9} - \frac{2yd}{3}\right] + (x-a)^2 + y^2$$

$$-\left[x^2 + \left(\frac{a+c}{3}\right)^2 - 2x\left(\frac{a+c}{3}\right) + y^2 + \frac{d^2}{9} - \frac{2yd}{3}\right] + (x-c)^2 + (y-d)^2$$

$$-\left[x^2 + \left(\frac{a+c}{3}\right)^2 - 2x\left(\frac{a+c}{3}\right) + y^2 + \frac{d^2}{9} - \frac{2yd}{3}\right]$$

$$= x^2 + y + x^2 + x^2 + a^2 - 2ax + y^2 + x^2 + c^2 - 2xc + y^2 + d^2 - 2yd - 3$$

$$\left[x^2 + \left(\frac{a+c}{3}\right)^2 - 2x\left(\frac{a+c}{3}\right) + y^2 + \frac{d^2}{9} - \frac{2yd}{3}\right]$$

$$= \cancel{3x^2} + \cancel{3y^2} + a^2 + c^2 + d^2 - 2ax - 2xc - 2yd - \cancel{3x^2} - \frac{(a+c)^2}{3} + 2x(a+c) - \cancel{3y^2} - \frac{d^2}{3} + 2yd$$

$$= a^2 + c^2 + d^2 - \cancel{2ax} - \cancel{2xc} - \cancel{2yd} - \frac{a^2 + c^2 + 2ac}{3} + \cancel{2ax} + \cancel{2cx} - \frac{d^2}{3} + \cancel{2yd}$$

$$= \frac{3a^2 + 3c^2 + 3d^2 - a^2 - c^2 - 2ac - d^2}{3} = \frac{2a^2 + 2c^2 + 2d^2 - 2ac}{3} = L.H.S$$

Solving R.H.S

$GA^2 + GB^2 + GC^2$

$$GA^2 = \left(\frac{a+c}{3}\right)^2 + \left(\frac{d}{3}\right)^2 = \frac{a^2 + c^2 + 2ac}{9} + \frac{d^2}{9}$$

$$GC^2 = \left(\frac{a+c}{3} = a\right)^2 + \left(\frac{d}{3}\right)^2 = \left(\frac{c-2a}{3}\right)^2 + \left(\frac{d}{3}\right)^2$$

$$= \frac{a^2 + 4c^2 - 4ca}{9} + \frac{4d^2}{9}$$

$$GB^2 = \left(\frac{a+c}{3} - a\right)^2 + \left(\frac{d}{3}\right)^2 = \left(\frac{c-2a}{3}\right)^2 + \left(\frac{d}{3}\right)^2$$

$$= \frac{c^2 + 4a^2 - 4ac}{9} + \frac{d^2}{9}$$

$$GA^2 + GB^2 + GC^2 = \frac{a^2 + c^2 + 2ac}{9} + \frac{d^2}{9} + \frac{a^2 + 4c^2 - 4ac}{9} + \frac{4d^2}{9} + \frac{c^2 + 4a^2 - 4ac}{9} + \frac{d^2}{9}$$

$$= \frac{a^2 + c^2 + 2ac + d^2 + a^2 + 4c^2 - 4ac + 4d^2 + c^2 + 4a^2 - 4ac + d^2}{9}$$

$$= \frac{6a^2 + 6c^2 + 6d^2 + 6ac}{9} = \frac{2a^2 + 2c^2 + 2d^2 + 2ac}{3}$$

$\therefore L.H.S = R.H.S$