RD SHARMA
Solutions
Class 10 Maths
Chapter 10
Ex10. 2

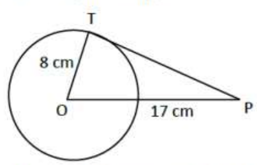
1. If PT is a tangent at T to a circle whose center is O and OP = 17 cm, OT = 8 cm. Find the length of tangent segment PT.

Sol:

OT = radius = 8cm

OP = 17cm

PT = length of tangent = ?



T is point of contact. We know that at point of contact tangent and radius are perpendicular.

∴ OTP is right angled triangle ∠OTP = 90°, from Pythagoras theorem $OT^2 + PT^2 = OP^2$

$8^2 + PT^2 = 17^2$

PT $\sqrt{17^2 - 8^2} = \sqrt{289 - 64}$

$= \sqrt{225} = 15cm$

∴ PT = length of tangent = 15 cm.

2. Find the length of a tangent drawn to a circle with radius 5cm, from a point 13 cm from the center of the circle.

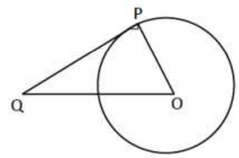
Sol:

Consider a circle with center O.

OP = radius = 5 cm.

A tangent is drawn at point P, such that line through O intersects it at Q, OB = 13cm.

Length of tangent PQ = ?



A + P, we know that tangent and radius are perpendicular.

ΔOPQ is right angled triangle, $\angle OPQ = 90°$

$By\ pythagoras\ theorem, OQ^2 = OP^2 + PQ^2$

$\Rightarrow 13^2 = 5^2 + PQ^2$

$\Rightarrow PQ^2 = 169 - 25 = 144$

$\Rightarrow PQ = \sqrt{144} = 12cm$

Length of tangent = 12 cm

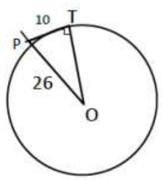
3. A point P is 26 cm away from O of circle and the length PT of the tangent drawn from P to the circle is 10 cm. Find the radius of the circle.

Sol:

Given OP = 26 cm

PT = length of tangent = 10cm

radius = OT = ?



At point of contact, radius and tangent are perpendicular $\angle OTP = 90°$, ΔOTP is right angled triangle.

By Pythagoras theorem, $OP^2 = OT^2 + PT^2$

$26^2 = OT^2 + 10^2$

$OT^k = \left(\sqrt{676 - 100}\right)^k$

$OT = \sqrt{576}$

= 24 cm

OT = length of tangent = 24 cm

4. If from any point on the common chord of two intersecting circles, tangents be drawn to circles, prove that they are equal.

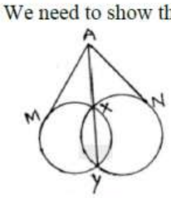
Sol:

Let the two circles intersect at points X and Y.

XY is the common chord.

Suppose 'A' is a point on the common chord and AM and AN be the tangents drawn A to the circle

We need to show that AM = AN.



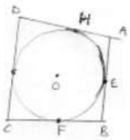
In order to prove the above relation, following property will be used.

"Let PT be a tangent to the circle from an external point P and a secant to the circle through P intersects the circle at points A and B, then $PT^2 = PA \times PB$"
Now AM is the tangent and AXY is a secant $\therefore AM^2 = AX \times AY \ldots\ldots (i)$
AN is a tangent and AXY is a secant $\therefore AN^2 = AX \times AY \ldots\ldots (ii)$
From (i) & (ii), we have $AM^2 = AN^2$
$\therefore$ AM = AN

5. If the quadrilateral sides touch the circle prove that sum of pair of opposite sides is equal to the sum of other pair.

Sol:

Consider a quadrilateral ABCD touching circle with center O at points E, F, G and H as in figure.



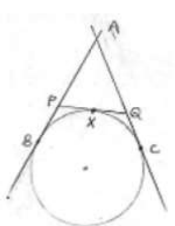
We know that
The tangents drawn from same external points to the circle are equal in length.

1. Consider tangents from point A [AM ⊥ AE]
   AH = AE …. (i)
2. From point B [EB & BF]
   BF = EB …. (ii)
3. From point C [CF & GC]
   FC = CG …. (iii)
4. From point D [DG & DH]
   DH = DG …. (iv)
   Adding (i), (ii), (iii), & (iv)
   (AH + BF + FC + DH) = [(AC + CB) + (CG + DG)]
   ⇒ (AH + DH) + (BF + FC) = (AE + EB) + (CG + DG)
   ⇒ AD + BC = AB + DC [from fig.]
   Sum of one pair of opposite sides is equal to other.

6. If AB, AC, PQ are tangents in Fig. and AB = 5cm find the perimeter of ΔAPQ.

Sol:

Perimeter of ΔAPQ, (P) = AP + AQ + PQ
= AP + AQ + (PX + QX)

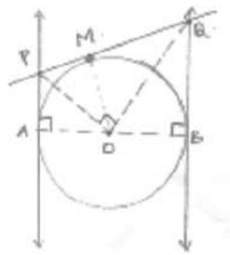


We know that
The two tangents drawn from external point to the circle are equal in length from point A,
AB = AC = 5 cm
From point P, PX = PB
From point Q, QX = QC
Perimeter (P) = AP + AQ + (PB + QC)
= (AP + PB) + (AQ + QC)
= AB + AC = 5 + 5
= 10 cms.

7. Prove that the intercept of a tangent between two parallel tangents to a circle subtends a right angle at center.

Sol:

Consider circle with center 'O' and has two parallel tangents through A & B at ends of diameter.



Let tangents through M intersects the tangents parallel at P and Q required to prove is that ∠POQ = 90°.

From fig. it is clear that ABQP is a quadrilateral

∠A + ∠B = 90° + 90° = 180° [At point of contact tangent & radius are perpendicular]

∠A + ∠B + ∠P + ∠Q = 360° [Angle sum property]

∠P + ∠Q = 360°−180° = 180° …..(i)

At P & Q ∠APO = ∠OPQ = $\frac{1}{2}\angle P$

∠BQO = ∠PQO = $\frac{1}{2}\angle Q$ in (i)

2∠OPQ + 2 ∠PQO = 180°

∠OPQ + ∠PQO = 90° …. (ii)

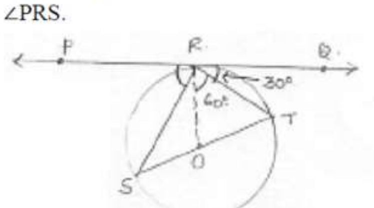
In ΔOPQ, ∠OPQ + ∠PQO + ∠POQ = 180° [Angle sum property]

90° + ∠POQ = 180° [from (ii)]

∠POQ = 180° − 90° = 90°

∴ ∠POQ = 90°

8. In Fig below, PQ is tangent at point R of the circle with center O. If ∠TRQ = 30°. Find ∠PRS.



Sol:

Given ∠TRQ = 30°.

At point R, OR ⊥ RQ.

∠ORQ = 90°

⇒ ∠TRQ + ∠ORT = 90°

⇒ ∠ORT = 90° − 30° = 60°

ST is diameter, ∠SRT = 90° [∵ Angle in semicircle = 90°]

∠ORT + ∠ SRO = 90°

∠SRO + ∠PRS = 90°

∠PRS = 90° − 30° = 60°

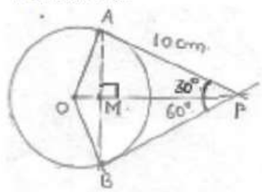
9. If PA and PB are tangents from an outside point P. such that PA = 10 cm and ∠APB = 60°. Find the length of chord AB.

Sol:

AP = 10 cm ∠APB = 60°

Represented in the figure

We know that



A line drawn from center to point from where external tangents are drawn divides or bisects the angle made by tangents at that point ∠APO = ∠OPB = $\frac{1}{2} \times 60° = 30°$

The chord AB will be bisected perpendicularly

∴ AB = 2AM

In ΔAMP,

$$\sin 30° = \frac{opp.side}{hypotenuse} = \frac{AM}{AP}$$

$$AM = AP \sin 30°$$

$$= \frac{AP}{2} = \frac{10}{2} = 5cm$$

$$AP = 2\ AM = 10 \text{ cm}$$ ---- Method (i)

In ΔAMP, ∠AMP = 90°, ∠APM = 30°

$$\angle AMP + \angle APM + \angle MAP = 180°$$

$$90° + 30° + \angle MAP = 180°$$

$$\angle MAP = 180°$$

In ΔPAB, ∠MAP = ∠BAP = 60°, ∠APB = 60°

We also get, ∠PBA = 60°

∴ΔPAB is equilateral triangle

AB = AP = 10 cm. -----Method (ii)